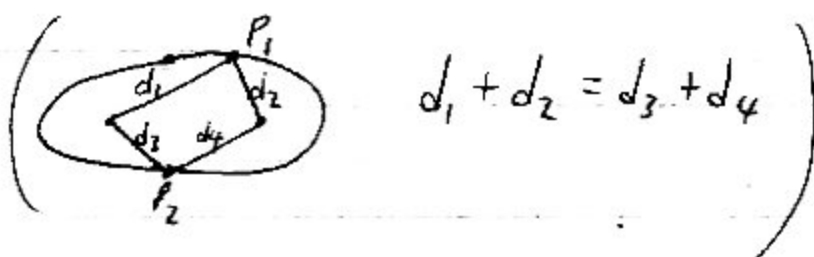
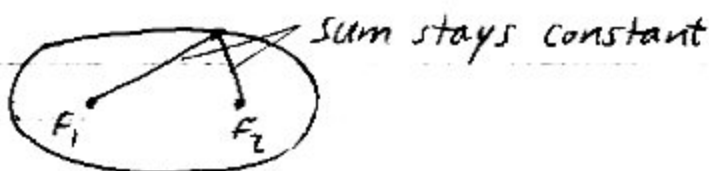


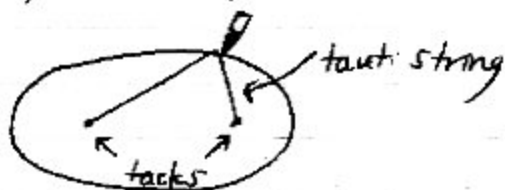
10.3: ELLIPSES

Ⓐ Locus Definition (of an Ellipse)

Idea Two fixed points (foci).

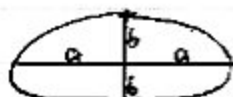
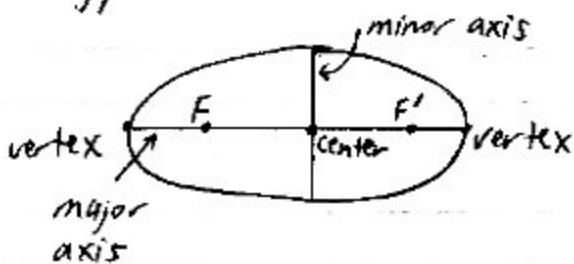


Constructing an Ellipse

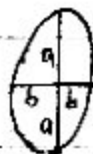


rubber band (elastix)
string stays same length

Ⓑ Terminology



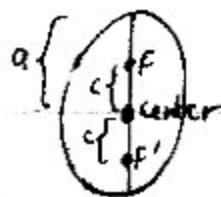
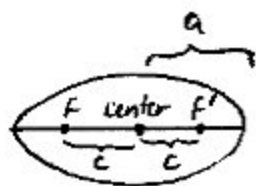
major axis = $2a$
minor axis = $2b$



We'll assume $0 < b < a$
(i.e., $a > b$
with "a")

If $a = b \rightarrow$ circle \oplus

$a = b = r$



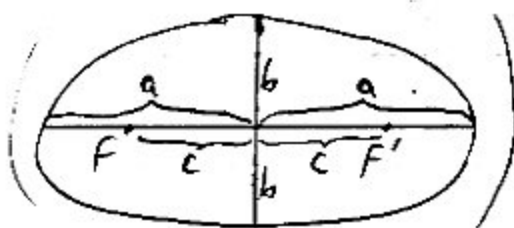
$$c^2 = a^2 - b^2$$

Notice: $c < a$

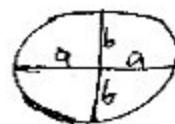
We'll assume $0 < b < a$.
 i.e., $a > b$
 both "+"
 If $a = b \rightarrow$ circle \oplus

③ If the Center is $(0,0)$

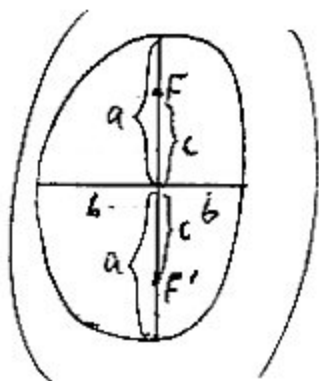
① $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 larger denom. under x^2



"x-long"



② $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
 larger denom. under y^2



"y-long"



① If the center is (h, k)

$$\textcircled{1} \rightarrow \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \textcircled{(h, k)}$$

$$\textcircled{2} \rightarrow \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad \textcircled{(h, k)}$$

To quote
Bernie Hopler

⑤ Eccentricity "e"

$$e = \frac{c}{a}$$

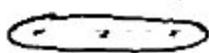


Circle

$$e=0$$



$$e=0.3$$



$$e=0.8$$

$$\text{Ellipse: } \begin{pmatrix} 0 < c < a \\ 0 < \frac{c}{a} < 1 \\ 0 < e < 1 \end{pmatrix}$$

⑥ Ex (#16)

Graph $9x^2 + 4y^2 - 54x + 40y + 37 = 0$
Find the center, vertices, foci, and e.

Group terms.

"Like" $\frac{x^2}{16} + \frac{y^2}{36} = 1$ (0 y-long)

$b^2 = 16$ $a^2 = 36$ (larger denom.)
 $b = 4$ $a = 6$ (take "+" root)

Find c

$$c^2 = a^2 - b^2$$

$$= 36 - 16$$

$$= 20$$

$$c = \sqrt{20}$$

$$= 2\sqrt{5}$$

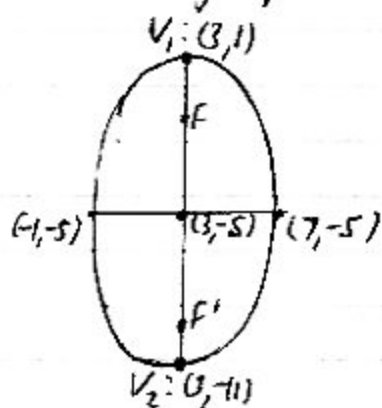
Find e

$$e = \frac{c}{a}$$

$$= \frac{2\sqrt{5}}{6}$$

$$= \frac{\sqrt{5}}{3} \approx 0.745$$

Sketch the graph



$$a = 6$$

$$b = 4$$

$$c = 2\sqrt{5}$$

"y stuff"
has larger denom. (a^2),
so y-long 0

$$F: (3, -5 + 2\sqrt{5})$$

$$F': (3, -5 - 2\sqrt{5})$$


can wait...

© Applications

Reflecting Property
Pool Table

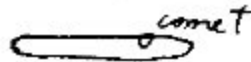
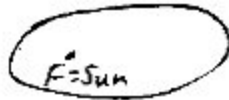


Statuary Hall in U.S. Capitol
J.Q. Adams eavesdropped.

Lithotripsy - destroying kidney stones 

what happens
surrounding
issue

Kepler's Laws of Planetary Motion (1609)



(can have
parabolic,
hyperbolic
orbit)

equal areas
in equal times

small effect
on temp. time!

Keplerian
orbit to
more calendar
not accurate
w/ general relativity

Larson 646

Comets desc.
before 1970
48% parabolic
40% ell.
12% hyp.

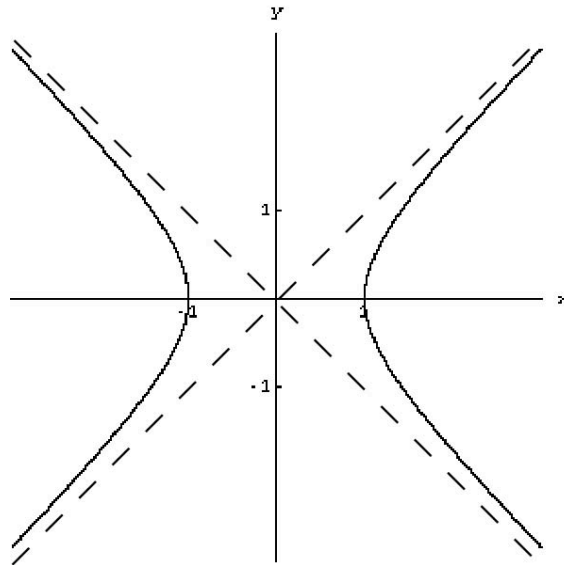
Halley $e = 0.97$
Kohoutek: $e = 0.999725$

SECTION 10.4: HYPERBOLAS

Hyperbolas have two branches. (They may be considered as part of a single branch, if you allow the branches to pass through the point at infinity. Think of a baseball. Don't worry about this for now!)

Technical Note: The locus definition of hyperbolas is similar to the one for ellipses, except that, instead of the sum of the distances to the two foci being kept constant, it is the absolute value of the difference of the distances.

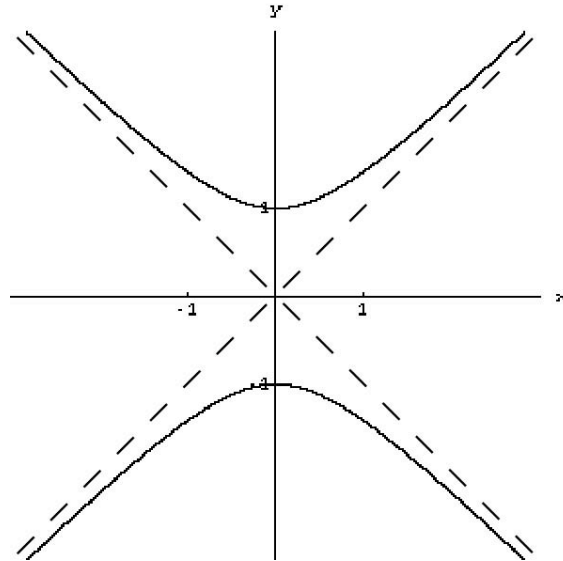
The graph of $x^2 - y^2 = 1$ is below:



The equation confirms that ± 1 are x -intercepts of the graph (plug in $y = 0$), but there are no y -intercepts (plug in $x = 0$ to see why).

The dashed lines are the asymptotes of the hyperbola. The two branches approach those lines but never cross them.

The graph of $y^2 - x^2 = 1$ is below:



The equation confirms that ± 1 are y -intercepts of the graph (plug in $x = 0$), but there are no x -intercepts (plug in $y = 0$ to see why).

It helps to remember that:

- The graph of $x^2 - y^2 = 1$ “opens” along the x -axis, while
- The graph of $y^2 - x^2 = 1$ “opens” along the y -axis.