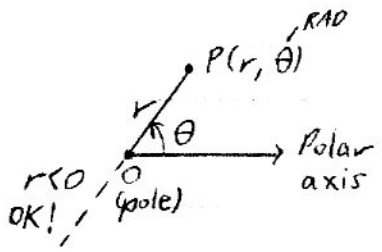
James Bern.
intro PCs in 1691
but Newton
may have used 1st
Calc 361: 1st suggested
by V (≈1671)

10.8: POLAR COORDS (PCs)

(A) PCs

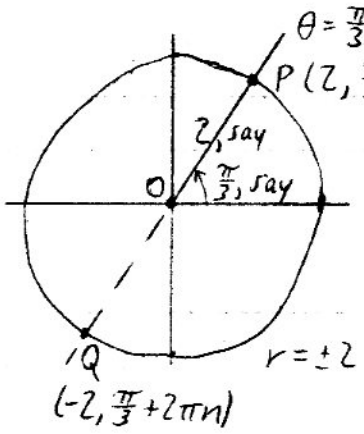
c: I've buried a treasure chest. You can assume 2 ?s.
Rect (Cart.) → ?
Michael Colton fix.

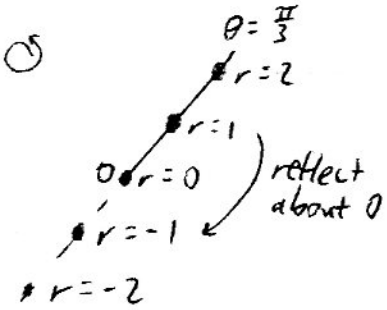


Pole O: $(0, \theta)$ ~~any~~ angle

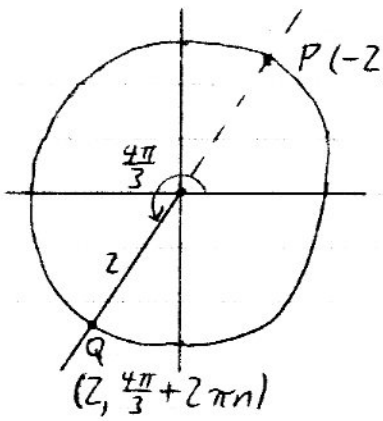
P has ∞ many PC reps.

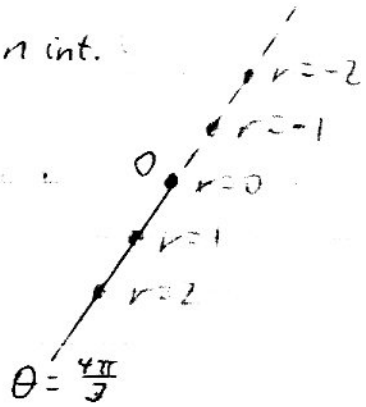
Hard: I graph $r = 2$? What pts have r-coord = 2? What figure do they form.





or





(B) Graph of a Polar Eq.

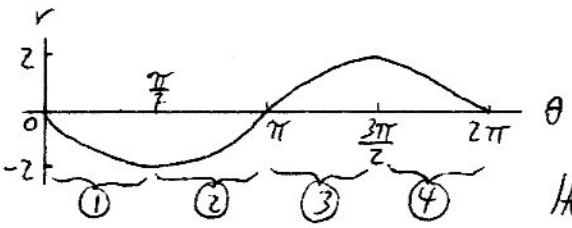
consists of all pts. (r, θ) satisfy eq.

Usual form: r or $r^2 = f(\theta)$

Ex $r = -2\sin\theta$

Stewart 673 "in Cartesian coords"

Graph r vs. θ as Cartesian/rectangular coords.



Be careful about calling these Q (r<0, interval θ)

r: $0 \to -2 \to 0 \to 2 \to 0$

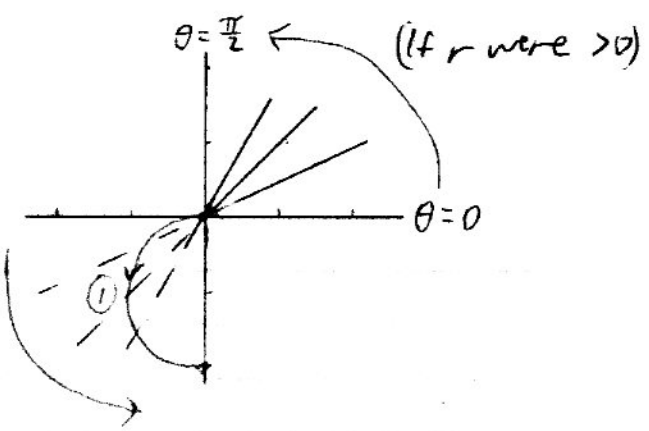
Here, "sectors" correspond to quadrants, but be careful!

or Table:

θ	r
0	$-2\sin(0) = 0$
⋮	

If $r > 0$, we'd be going thru QI

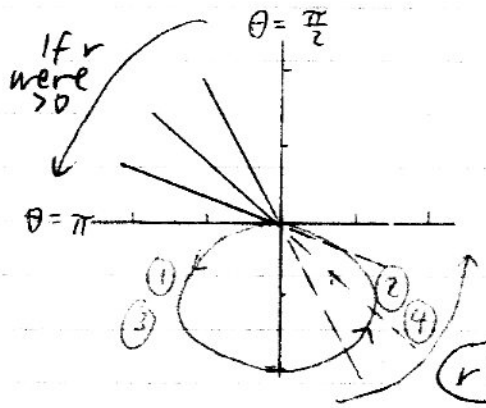
Do ①



Trust me: it's a half-circle Recognize basic forms

$r: 0 \to -2$

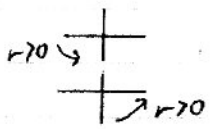
Do ②



Circle → We'll show how you can prove this later.

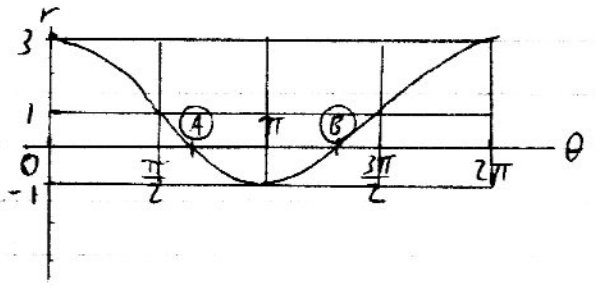
r<0
r: -2 → 0

③ retraces ①
④ ②



Ex $r = 1 + 2\cos\theta$

Note:
$-2 \le 2\cos\theta \le 2$
$-1 \le 1 + 2\cos\theta \le 3$



① ② ③ ④ ⑤ ⑥

r: 3 → 1 → 0 → -1 → 0 → 1 → 3

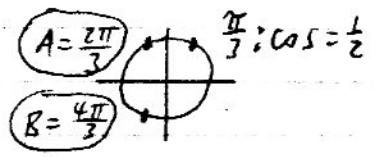
Sectors divided by:
- quadrants
- ↗ vs. ↘
- + vs. −

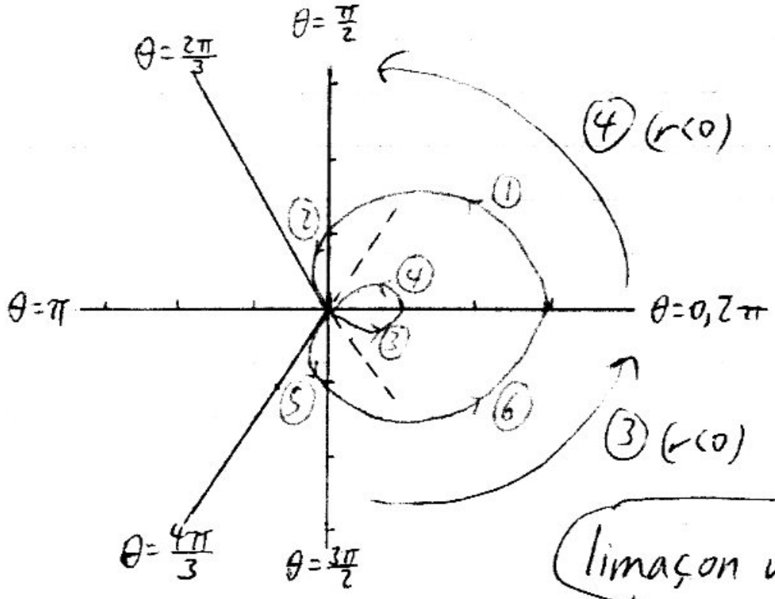
Find A, B

$0 = 1 + 2\cos\theta$

$\cos\theta = -\frac{1}{2}$

Discuss





limaçon w/a loop
snail

Lee-muh-SOH"
Webster: locus set
fr: snail
escargot?
'prepared snail

Stewart 677
$r = 1 + c\sin\theta$
Me
$r = 1 + a\cos\theta$
$a = 1$ cardioid
$a \neq 0, 1$: limaçon
$0 < a < 1$: no loop
$\frac{1}{2} < a < 1$: dimple

$0 < a \leq \frac{1}{2}$
no dimple
oval (a.k.a. 677: "convex")
$a \to 0$, $\to$ circle
$r = 1 + \cos\theta$

$r = 1 + \cos\theta$

cardioid – $\overset{\approx}{\text{"heart"}}$ (butt?)

The evolution of $r = 1 + a\cos\theta$

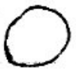
$r = 1 + 0.7\cos\theta$

dimpled limaçon

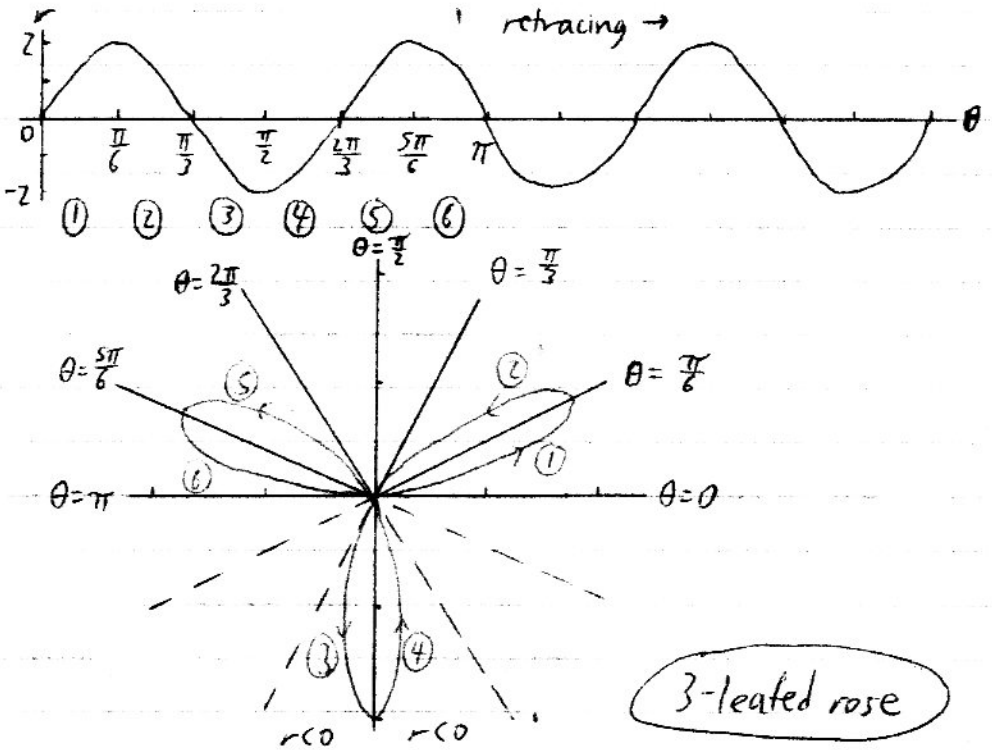
 $r = 1 + 0.5\cos\theta$ (loses dimple)

loses dimple
still limaçons

$r = 1$

 Circle!

Ex $r = 2\sin(3\theta)$

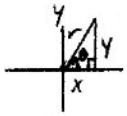


$r = a \begin{matrix}\sin\\ \cos\end{matrix}(n\theta)$

$a \neq 0$ if $n = 3, 5, 7, \ldots \Rightarrow n$ leaves

if $n = 2, 4, 6, \ldots \Rightarrow 2n$ leaves

© Polar Eq. ⇔ Rect. Eq.



$$r^2 = x^2 + y^2$$
$$\tan\theta = \frac{y}{x},\ x \neq 0$$
← Watch quadrant?

$\cos\theta = \frac{x}{r} \Rightarrow$
$\sin\theta = \frac{y}{r} \Rightarrow$

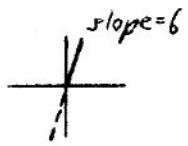
$$x = r\cos\theta$$
$$y = r\sin\theta$$
even if $r < 0$

Ex Find a polar eq. w/same graph as $y = 6x$

$\frac{y}{x} = 6$ Also, (0,0)

$\tan\theta = 6$

$\theta = \tan^{-1} 6$



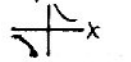
Ex Find a rect. eq. w/same graph as $r^2 \sin(2\theta) = 4$, and graph it.

$r^2(2\sin\theta\cos\theta) = 4$

$2\underbrace{(r\sin\theta)}_{=y}\underbrace{(r\cos\theta)}_{=x} = 4$

$2xy = 4$

$xy = 2$ or $y = \frac{2}{x}$



Ex (again) $r = -2\sin\theta$

$r^2 = -2r\sin\theta$

$x^2 + y^2 = -2y$

$x^2 + y^2 + 2y = 0$

$x^2 + (y^2 + 2y + 1) = 1$

$x^2 + (y+1)^2 = 1$

✓ when $r = 0$ OK?

Circle w/center: (0, −1)
radius = 1