6.7: MOMENTS and CENTERS OF MASS

(A) One-Dimensional Setting

relevant to your everyday lives

Ex

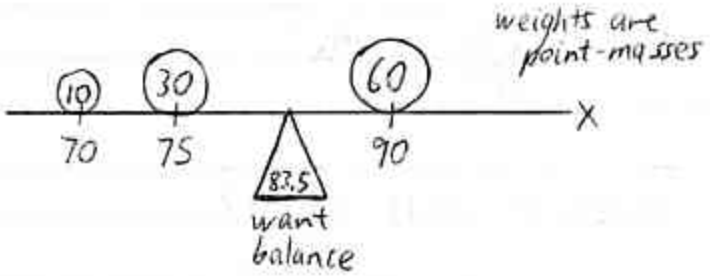
Test #k	Weight (% of class grade) (m_k)	Your score (x_k)
Test 1	10	70
Test 2	30	75
Test 3	60	90
(n=3 tests)	$\sum_{k=1}^{3} m_k = 100$	

quiz
mid
final
no HW-lucky!

what do you want to know?

Find your weighted % for the class.

not boulders



mean

$\bar{x}$ = center of mass

$$= \frac{\sum_{k=1}^{3} m_k x_k}{\sum_{k=1}^{3} m_k}$$

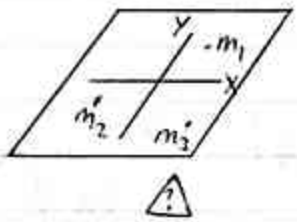
← moment "M_0" about the origin
← "m", the total mass

The quiz was worth 10%.

$$= \frac{(10)(70) + (30)(75) + (60)(90)}{100} \rightarrow \text{Press } \boxed{=}$$

$$= \boxed{83.5}$$

(B) Two-Dimensional Setting



m_1 is at (x_1, y_1)

For $k = 1, 2, \ldots, n$:
The point-mass m_k is at (x_k, y_k).

Let $m = \sum_{k=1}^{n} m_k$
= total mass

$$\bar{x} = \frac{\sum_{k=1}^{n} m_k x_k}{m} \leftarrow \text{"}M_y\text{"}$$, the moment about y-axis

You're standing somewhere on the y-axis and you're looking around for $\bar{x}$.

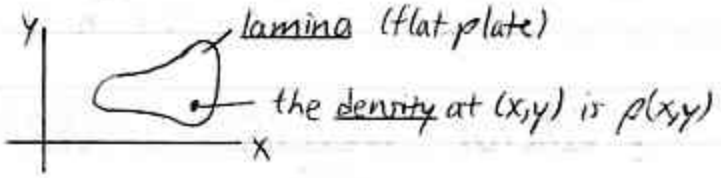
$$\bar{y} = \frac{\sum_{k=1}^{n} m_k y_k}{m} \leftarrow \text{"}M_x\text{"}$$

Center of mass: $(\bar{x}, \bar{y})$

(C) The Centroid of a Region

p. 426

Calc III

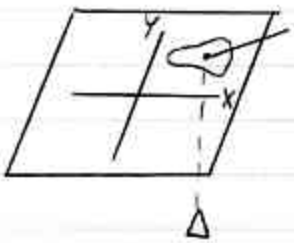


(in III, use δ)

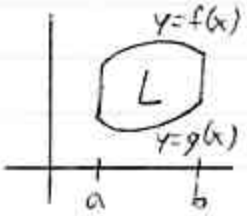
Calc I

Assume $\rho(x,y) = 1$ throughout the lamina.
homogeneous
(constant density)

Then, m (total mass) $= A$ (area).



$(\bar{x},\bar{y})$ is the center of mass of the laming, or the centroid of the region



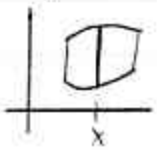
Find the centroid $(\bar{x},\bar{y})$ of L.

m = mass of L
= area
$= \int_a^b [f(x) - g(x)]\,dx$
(top − bottom)

$\bar{x} = \frac{M_y}{m}$

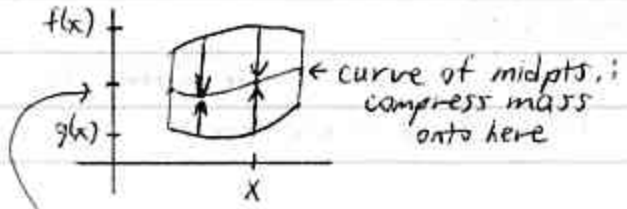
adding there x-weight at x

where $M_y = \int_a^b x\,\underbrace{[f(x)-g(x)]}_{\text{weight at } x}\,dx$

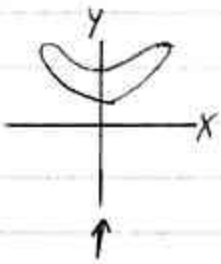


$\bar{y} = \frac{M_x}{m}$

where $M_x = \int_a^b \underbrace{\frac{1}{2}[f(x)+g(x)]}_{\substack{y\text{-coord. on curve of midpts.} \\ \text{weighting } y\text{-coords.}}} \cdot \underbrace{[f(x)-g(x)]}_{\substack{\text{weight} \\ \text{at } x}}\,dx$



Read Exs. 3-6



$\bar{x}=0$ by symmetry