9.5: DEALING w/ QUADRATICS

Ex[(9.4)] $\int \frac{1}{\underbrace{x^2+3x-10}_{\text{Factor}}}\,dx$ ~~u-sub~~

$$= \int \frac{1}{(x+5)(x-2)}\,dx$$

$$= \frac{A}{x+5} + \frac{B}{x-2}$$

$$\vdots$$

$$A = -\tfrac{1}{7},\ B = \tfrac{1}{7}$$

$$= \int \frac{-\frac{1}{7}}{x+5}\,dx + \int \frac{\frac{1}{7}}{x-2}\,dx$$

Can use log props

$$= \boxed{-\tfrac{1}{7}\ln|x+5| + \tfrac{1}{7}\ln|x-2| + C}$$

Ex $\int \frac{x}{\underbrace{x^2+6x+10}_{\text{what if irred.?}}}\,dx$ ~~u-sub~~

Can't factor over reals

Complete the square (CTS):

$$x^2+6x+10 = (x^2+6x+9)+10-9$$
$$= (x+3)^2+1$$

$$= \int \frac{x}{(x+3)^2+1}\,dx$$

Let $u = x+3 \xRightarrow{\text{Solve for } x} x = u-3$

$du = dx$

$$= \int \frac{\overset{\text{split}}{u-3}}{u^2+1}\,du$$

$$= \underbrace{\int \frac{u}{u^2+1}\,du}_{1} - \underbrace{\int \frac{3}{u^2+1}\,du}_{2}$$

u,v reek of parts

1. $w = u^2+1$

$dw = 2u\,du \Rightarrow u\,du = \frac{1}{2}dw$

$$\frac{1}{2}\int \frac{dw}{w} = \frac{1}{2}\ln|w| + C_1$$

$$= \frac{1}{2}\ln(\underbrace{|u^2+1|}_{>0}) + C_1$$

$$= \frac{1}{2}\ln(u^2+1) + C_1$$

Go back to x!!

$$= \frac{1}{2}\ln[(x+3)^2+1] + C_1$$

should look familiar

$$= \frac{1}{2}\ln(x^2+6x+10) + C_1$$

2. $3\tan^{-1}u + C_2$

$$= 3\tan^{-1}(x+3) + C_2$$

Ds 3

$$= \boxed{\frac{1}{2}\ln(x^2+6x+10) - 3\tan^{-1}(x+3) + C}$$

We can factor $10x-x^2$, but...

Ex $\int \frac{1}{(10x-x^2)^{3/2}}\,dx$ ~~u-sub~~

Uh oh!

$\sqrt{\ } \Rightarrow$ probably no PFD

$$= \int \frac{1}{\left(\sqrt{10x-x^2}\right)^3}\,dx$$

(Aim for trig sub.)

$$\text{CTS: } 10x-x^2 = -x^2+10x$$
$$= -(x^2-10x \underbrace{+25}_{\text{really, } -25}) + 25$$
$$= -(x-5)^2+25$$
$$= 25-(x-5)^2$$

$$= \int \frac{1}{\left(\underbrace{\sqrt{25-(x-5)^2}}_{\text{Form: } \sqrt{a^2-u^2}}\right)^3}\,dx$$

sin, tan, or sec?

$u = a\sin\theta$

Sub: $x-5 = 5\sin\theta$

Can skip → $\left(x = 5\sin\theta + 5\right)$

$dx = 5\cos\theta\,d\theta$

$$= \int \frac{1}{\left(\sqrt{25-(5\sin\theta)^2}\right)^3}\cdot 5\cos\theta\,d\theta$$

$$\sqrt{25-(5\sin\theta)^2} = \sqrt{25-25\sin^2\theta}$$
$$= 5\sqrt{1-\sin^2\theta}$$
$$= 5\cos\theta$$

$$= \int \frac{1}{(5\cos\theta)^3} \cdot 5\cos\theta \, d\theta$$

$$= \int \frac{1}{(5\cos\theta)^2} \, d\theta$$

$$= \int \frac{1}{25\cos^2\theta} \, d\theta$$

Whose deriv. is $\sec^2\theta$?

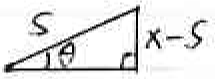
$$= \frac{1}{25} \int \sec^2\theta \, d\theta$$

$$= \frac{1}{25} \tan\theta + C$$

$\underline{\tan\theta = ?}$

$$x - 5 = 5\sin\theta$$

$$\sin\theta = \frac{x-5}{5}$$



Look familiar?

$(\sqrt{25-(x-5)^2}$, or$)$

$\sqrt{10x - x^2}$

Don't have to rat'lize denom.

$$\tan\theta = \frac{x-5}{\sqrt{10x-x^2}}$$

$$= \boxed{\frac{1}{25}\left(\frac{x-5}{\sqrt{10x-x^2}}\right) + C}$$