9.6: RATIONALIZING SUBS

Ex $\int \frac{x-2}{\sqrt{4x+1}}\,dx$

Not really a rat'lizing sub. Like 5th 2 for Ex 1 in book.

Method 1

Let $u = 4x+1$ $\xrightarrow{\text{Solve for } x}$ $x = \frac{u-1}{4}$

$du = 4\,dx \Longrightarrow dx = \frac{1}{4}\,du$

$$\int \frac{\frac{u-1}{4}-2}{\sqrt{u}} \cdot \frac{1}{4}\,du$$

$$= \frac{1}{4}\int \frac{4\left(\frac{u-1}{4}-2\right)}{4(\sqrt{u})}\,du$$

$$= \frac{1}{4}\int \frac{(u-1)-8}{4\sqrt{u}}\,du$$

$$= \frac{1}{16}\int \frac{u-9}{u^{1/2}}\,du$$

$$= \frac{1}{16}\int \left(u^{1/2} - 9u^{-1/2}\right)du$$

$$= \frac{1}{16}\left[\frac{u^{3/2}}{3/2} - 9\left(\frac{u^{1/2}}{1/2}\right)\right] + C$$

$$= \frac{1}{16}\left[\frac{2}{3}u^{3/2} - 18u^{1/2}\right] + C$$

Can factor out $\sqrt{4x+1}$

$$= \boxed{\frac{1}{24}(4x+1)^{3/2} - \frac{9}{8}\sqrt{4x+1} + C}$$

Method 2 (Rat'lizing Sub)

$$\int \frac{x-2}{\sqrt{4x+1}}\,dx$$

Let $u=\sqrt{4x+1}$

$u^2 = 4x+1 \Rightarrow x=\frac{u^2-1}{4}$

$2u\,du = 4\,dx \Rightarrow dx = \frac{1}{2}u\,du$

$$\int \frac{\frac{u^2-1}{4}-2}{u}\cdot\frac{1}{2}u\,du$$

$$= \frac{1}{2}\int \underbrace{\left(\frac{u^2-1}{4}-2\right)}_{\text{rational in } u}\,du$$

In fact poly

Easier

$$= \frac{1}{2}\left[\frac{1}{4}\int(u^2-1)\,du - \int 2\,du\right]$$

$$= \frac{1}{2}\left[\frac{1}{4}\left(\frac{u^3}{3}-u\right)-2u\right]+C$$

$$= \frac{1}{24}u^3 - \frac{1}{8}u - u + C$$

$$= \frac{1}{24}u^3 - \frac{9}{8}u + C$$

More complex setup, but we avoid frac exps

$$= \frac{1}{2}\int\left(\frac{u^2-1}{4}-\frac{8}{4}\right)du$$

$$= \frac{1}{2}\int\frac{u^2-9}{4}\,du$$

$$= \frac{1}{8}\int(u^2-9)\,du$$

$$= \frac{1}{8}\left[\frac{u^3}{3}-9u\right]+C$$

$$= \frac{1}{24}u^3 - \frac{9}{8}u + C$$

$$= \boxed{\frac{1}{24}\left(\sqrt{4x+1}\right)^3 - \frac{9}{8}\sqrt{4x+1}+C}$$

or $(4x+1)^{3/2}$

Can do 1, 3, 11 not 7

Ex $\int \frac{\sin(2x)}{3+\sin x}\,dx$

How before ⇒

Let $u = 3+\sin x \implies \sin x = u-3$

$du = \cos x\,dx$

(How do I pull $\cos x$ out of $\sin(2x)$?)

$$\int \frac{2\sin x\cos x}{3+\sin x}\,dx$$

$$= 2\int \frac{(u-3)\,du}{u}$$

$$= 2\int \left(1-\frac{3}{u}\right)du$$

$$= 2[u - 3\ln|u|] + C$$

$$= 2[(3+\sin x) - 3\ln|\underbrace{3+\sin x}_{>0}|] + C$$

C absorbs 6

Can do all but 7, 29

$$= \boxed{2\sin x - 6\ln(3+\sin x) + C}$$

Edwards 9.8 #7

Ex $\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx$

Can factor rationalized. or "conjugates" twice

How is $x^{1/2}$ related to $x^{1/4}$?

$\sqrt{x} = x^{1/2}$

$\sqrt[4]{x} = x^{1/4}$

LCD = 4

Let $u = x^{1/4} \Rightarrow u^4 = x$

$4u^3\,du = dx$

$u^2 = x^{1/2}$

Can I simp this frac?

$$\int \frac{1-u^2}{1+u} \cdot 4u^3\,du$$

$$= \int \frac{(1+u)(1-u)}{1+u} \cdot 4u^3\,du$$

$$= 4\int (u^3 - u^4)\,du$$

$$= 4\left[\frac{u^4}{4} - \frac{u^5}{5}\right] + C$$

$$= u^4 - \frac{4}{5}u^5 + C$$

$$= x - \frac{4}{5}(x^{1/4})^5 + C$$

Can do all but 29

$$= \boxed{x - \frac{4}{5}x^{5/4} + C}$$

Ex (#29) $\int \underbrace{\frac{1}{1+\sin x + \cos x}}_{\text{rational in } \sin x, \cos x}\,dx$

~~standard u-sub~~

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Sub: $\sin x = \frac{2u}{1+u^2}$, $\cos x = \frac{1-u^2}{1+u^2}$, $dx = \frac{2}{1+u^2}\,du$ } I'll give as needed. $\Rightarrow \int (\text{rat'l in } u)\,du$

where $u = \tan\left(\frac{x}{2}\right)$ } Know.