7.6 LAWS OF GROWTH AND DECAY and 19.1 SEPARABLE DIFFERENTIAL EQS.

Exs Pop. growth, Cont. Compounded Interest
Radioactive decay

If the rate of change of f (differentiable) with respect to x is proportional to the value of f, what can f be?

$$y = f(x)$$

Solve: $y' = ky$

k is like a constant of proportionality.

Note that $y=0$ is a sol'n.
For now, assume $y > 0$ "where we care."

If $k > 0 \Rightarrow y' > 0 \Rightarrow$ growth in y
$\quad <\qquad\quad <\qquad$ decay

$$\frac{dy}{dx} = ky$$

Separation of variables

$$\frac{dy}{y} = k\,dx \quad \leftarrow \text{Differential form}$$

$$\underbrace{\frac{dy}{y}}_{\begin{pmatrix}\text{Stuff}\\ \text{in } y, dy\end{pmatrix}} = \underbrace{k\,dx}_{\begin{pmatrix}\text{Stuff}\\ \text{in } x, dx\end{pmatrix}}$$

We shouldn't say, "We multiplied by dx." Whatever....

$$\int \frac{dy}{y} = \int k\,dx$$

$$\ln|y| = kx + C$$

Can remove (y>0)

$$e^{\ln y} = e^{kx + C}$$

$$y = e^{kx} \underbrace{e^{C}}_{= D}$$

$$y = De^{kx}$$

or $f(x)$

$$\text{If } x = 0 \Rightarrow f(0) = D\underbrace{e^{k(0)}}_{=1}$$

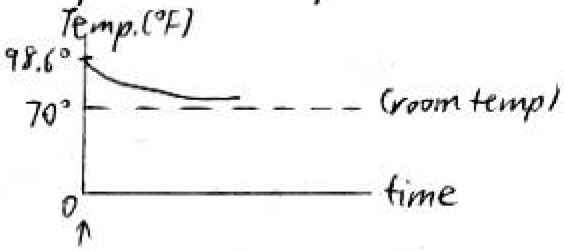
$$\text{"}y_0\text{"} = D$$

$$\boxed{y = y_0 e^{kx}}$$

Ex 3 (pp. 418-9) Newton's Law of Cooling

Dead professor problem

98.2°?
40°?



Murder
most foul!

Ex Solve $\cos x \cot x \frac{dy}{dx} + 7y = 0$

Assume $y \neq 0$ (Note that $y=0$ is a sol'n.)
$\cos x, \cot x \neq 0$, are defined "where we care"

Differential form:

$$\cos x \cot x \, dy + 7y \, dx = 0$$

$$\cos x \cot x \, dy = -7y \, dx$$

$$\int \frac{dy}{y} = \int \frac{-7}{\cos x \cot x} \, dx$$

$$\ln|y| = \int -7 \sec x \tan x \, dx$$

$$\ln|y| = -7 \sec x + C$$

$$e^{\ln|y|} = e^{-7\sec x + C}$$

$$|y| = e^{-7\sec x} e^{C}$$

$$y = \underbrace{\pm e^{C}}_{\neq 0} e^{-7\sec x}$$

$$y = De^{-7\sec x}$$

Note: $D \neq 0$, though
$D = 0 \Rightarrow y = 0$ (sol'n)

Recommended: 19.1, #11

(6.6) WORK

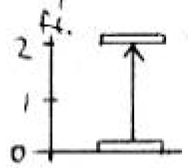
If constant force F $\longrightarrow$ (object) $\longrightarrow$ moves a distance of d units

Work done $W = Fd$

force distance

Ex A 50-pound rod is lifted 2 feet.

50 pounds is the magnitude of the force required to lift the rod.

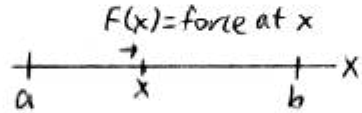


$$W = Fd$$
$$= (50 \text{ pounds})(2 \text{ feet})$$
$$= 100 \text{ foot-pounds}$$

Metric: 1 Newton-meter (N-m), or
1 joule (J)

≈ 0.74 foot-pounds

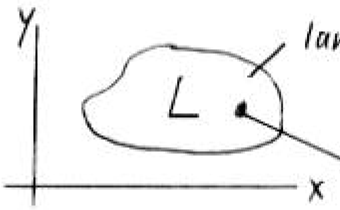
Ex F: variable force (continuous)



Work done in moving an object from $x=a$ to $x=b$ along —x =

$$W = \int_a^b F(x)\,dx$$

(6.7) MOMENTS and CENTERS OF MASS



lamina (flat plate) consists of point masses. Really, there has to be thickness.

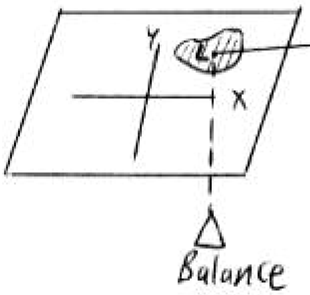
the density at (x,y) is $\rho(x,y)$.
Here, it's mass per unit area, really, volume normally

Larson 458, 6ed

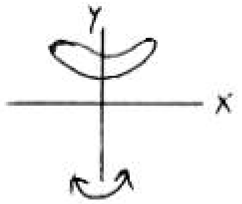
For now, assume $\rho(x,y)=1$ throughout L.

constant, so <u>homogeneous</u>

Then, mass = Area

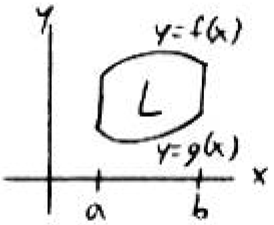


The centroid of the region $(\bar{x},\bar{y})$ is the center of mass of L.



$\bar{x}=0$ by symmetry

I won't test you on this page, but take a look!



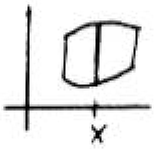
Find the centroid $(\bar{x}, \bar{y})$ of L.

$$m = \text{mass of } L = \text{area} = \int_a^b [\underbrace{f(x)}_{\text{top}} - \underbrace{g(x)}_{\text{bottom}}]\,dx$$

$$\bar{x} = \frac{M_y}{m}$$ ← Moment of L about the y-axis } like an average

"adding" there: x · weight at x

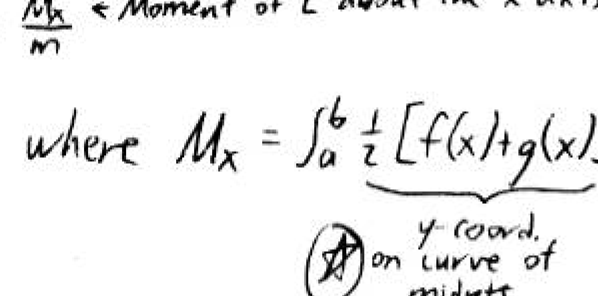
$$\text{where } M_y = \int_a^b x\,\underbrace{[f(x) - g(x)]}_{\text{weight at } x}\,dx$$



$$\bar{y} = \frac{M_x}{m}$$ ← Moment of L about the x-axis

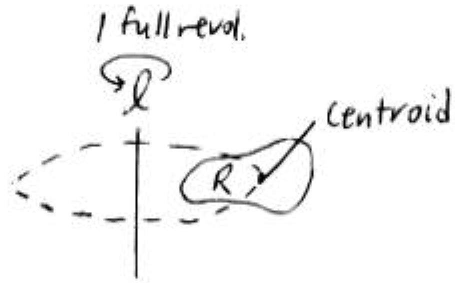
$$\text{where } M_x = \int_a^b \underbrace{\underbrace{\tfrac{1}{2}[f(x) + g(x)]}_{\text{y-coord. on curve of midpts.}} \cdot \underbrace{[f(x) - g(x)]}_{\text{weight at } x}}_{\text{weighting y-coords.}}\,dx$$

Why ☆?



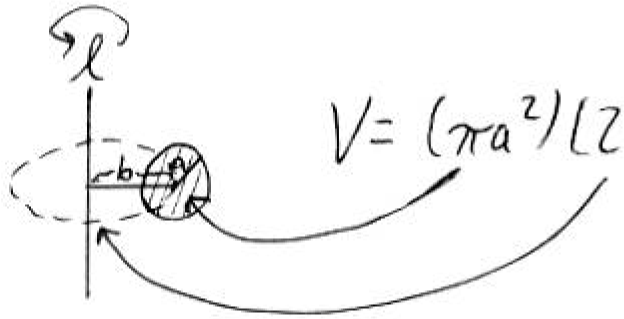
average $= \frac{1}{2}(f(x) + g(x))$

Theorem of Pappus (Cool!)



Volume of resulting solid
$= (\text{Area of } R)\left(\begin{array}{c}\text{distance}\\ \text{traveled by}\\ \text{centroid}\end{array}\right)$

Ex (Donut/Torus)



$$V = (\pi a^2)(2\pi b) = 2\pi^2 a^2 b$$