Larson 662
Many 17c, 18c
mathems:
Gregory, Newton,
John/James
Bernoulli, Leibniz,
Euler, Lagrange,
Fourier
Wallis

11.7: POWER Σ REPRESENTATION OF f(x)

Ⓐ Exs (Geometric Template)

If you plug in $x = 0.1$

Ex $\underbrace{1 + x + x^2 + \dots + x^n + \dots}_{\text{a power } \Sigma \text{ rep. for } f(x) = \frac{1}{1-x} \text{ on } (-1,1)} = \overbrace{\frac{1}{1-x}}^{\text{sum of } \Sigma}$ if $|x|<1$ (else Ⓓ)

a power Σ rep. for
$f(x) = \frac{1}{1-x}$ on $\underbrace{(-1,1)}_{I = \text{Domain of } f}$

For every x in Dom, sum of series is a real #
Outside I, the sum is und., too

I = Domain of f

$$\boxed{\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \ , \ |x|<1}$$

If know ctr. ⇒ unique power Σ
Jeopardy

Larson 616-7: Taylor for rat'l func: Long ÷
$3-2x\overline{)1}$
PFD
$I = \cap I_i$

(Find a power Σ w/this sum.)

Ex (#4) Find a power Σ rep. for $f(x) = \frac{1}{3-2x}$

Rewrite as $\underbrace{\binom{\text{const.}}{\text{or } x^k}}_{}\cdot\frac{1}{1-☆}$
(Easy to distribute ⇒ still have power Σ)

$$f(x) = \frac{1}{3-2x}$$
↖ want "1"

$$= \frac{1}{3(1-\frac{2}{3}x)}$$

$$= \frac{1}{3}\cdot\frac{1}{1-\frac{2}{3}x}$$
☆

$$\frac{1}{1-☆} = \sum_{n=0}^{\infty} ☆^n \ , \ |☆|<1$$

$$= \frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{2}{3}x\right)^n, \quad \left|\frac{2}{3}x\right| < 1$$

$$= \frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n x^n, \quad \frac{2}{3}|x| < 1$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}} x^n, \quad |x| < \frac{3}{2}}$$

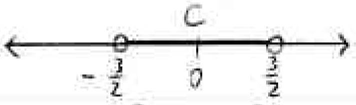
$$R = \frac{3}{2}$$

Need another I if develop another Σ. Larson 615

$\frac{1}{1+x}$ ctr. at -1

$= \frac{1}{2} \cdot \frac{1}{2-(x+1)}$

I:



Know (1) bec. Geom. Σ

$$\left(-\frac{3}{2}, \frac{3}{2}\right)$$

Like #8, but replace − w/ +

Ex

$$f(x) = \frac{x^3}{4+x^3}$$

Want "1"

$$= \frac{x^3}{4} \cdot \frac{1}{1 + \frac{x^3}{4}}$$

Want "−"

$$= \frac{x^3}{4} \cdot \frac{1}{1 - \left(-\frac{x^3}{4}\right)}$$

☆

$$= \frac{x^3}{4} \cdot \sum_{n=0}^{\infty} \left(-\frac{x^3}{4}\right)^n, \quad \left|-\frac{x^3}{4}\right| < 1$$

$$= \frac{x^3}{4} \sum_{n=0}^{\infty} (-1)^n \frac{x^{3n}}{4^n}, \quad \frac{|x^3|}{4} < 1$$

$|x^3| = |x \cdot x \cdot x| = |x||x||x| = |x|^3$

$\sqrt[3]{\ }$ monotonically ↗; even, odd irrelevant for 1-1

Do 1-7 odd, 9 if assign (No) except 16, 36

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{4^{n+1}} \cdot x^{3n+3}, \qquad |x^3| < 4, \quad |x|^3 < 4, \quad |x| < \sqrt[3]{4}$$

$$R = \sqrt[3]{4}$$

$$I = (-\sqrt[3]{4}, \sqrt[3]{4})$$

bec. Geom. Σ

(B) Finding $f'(x)$ Using Power Σ (Important in Diff. Eqs.!!)

Review Ex

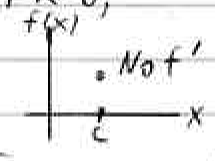
Always true if have finite # of terms, differentiating

$$\underbrace{D_x}_{\text{linear operator}} (x^2 + 7x^3) = D_x(x^2) + D_x(7x^3) = 2x + 7(3x^2) = 2x + 21x^2$$

If graph of f just consists of 1 pt, Literally no point in talking about derivs./Σs

$\boxed{\text{Can } D_x \text{ a Power } \Sigma \text{ term-by-term (Need } R \neq 0)}$

(If $R = 0$, No f')

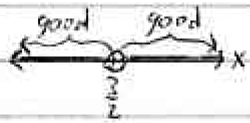


Ex (#4) $f(x) = \frac{1}{3-2x}$

Math 150 $f(x) = (3-2x)^{-1}$

or Recip. or Quot. rules

$$f'(x) = -(3-2x)^{-2}(-2) = \frac{2}{(3-2x)^2}$$



$x \neq \frac{3}{2}$

We found a power Σ rep. for $f(x)$ in Ⓐ.

$$\frac{1}{3-2x} = \sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}} x^n, \quad |x| < \frac{3}{2} \quad \left(R = \frac{3}{2}\right)$$

Note $\frac{1}{3-2x} = \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2 + \ldots$

$n=0$ term

$D_x\left(\frac{1}{3-2x}\right) = 0 + \underbrace{\frac{2}{9}}_{\text{Start w/}n=1} + \frac{8}{27}x + \ldots$

Works if the "$n=0$" term is a constant.

$$D_x\left(\frac{1}{3-2x}\right) = D_x\left(\sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}} x^n\right)$$

can switch for power Σ ((can D_x term-by-term)

$$= \sum_{n=1}^{\infty} D_x\left(\frac{2^n}{3^{n+1}} x^n\right)$$

indep. of x

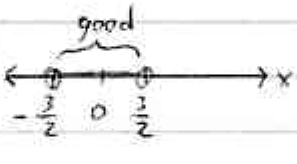
$$= \sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}} \cdot n x^{n-1}$$

$$= \sum_{n=1}^{\infty} \frac{n 2^n}{3^{n+1}} x^{n-1}$$

Again, $R = \frac{3}{2}$.
Turns out $I = \left(-\frac{3}{2}, \frac{3}{2}\right)$. for new Σ

Must ✓ endpts.
New Σ not geom.

Sum of new $\Sigma = \frac{2}{(3-2x)^2}$ Ⓐ
if x is in I.



$f'(x)$ as the Σ has a smaller domain!

Stewart: 11.9.36
$\sum \frac{\sin(n x)}{n^2}$ conv. $\forall x$
but $\sum f_n'$ div.
when $x = 2n\pi$,
n int.
Can't switch D_x Σ

$x = \frac{3}{2} \Rightarrow \sum \frac{2n}{9}$

$x = -\frac{3}{2} \Rightarrow \sum (-1)^{n-1} \frac{2}{9}$

$D_x \Sigma$ (geom) $\Rightarrow$ not geom.
not $a + a + a$...
In practice, use Ⓐ, but this is an easy Ex of D_x a Σ.
See Ⓔ

Can't use geom template early
$\frac{1}{3-2x} \frac{1}{3-2x} = \Sigma\Sigma$
CTS?

Carson 611:
For x in $(-d,d)$ $(R=d)$,
f cont., diff'e
$\Downarrow$
integrable

Ⓒ Finding $\int$s Using Power Σ

$$\boxed{\text{Can } \int \text{ a Power } \Sigma \text{ term-by-term (Need } R \neq 0)}$$

Non-shifted power Σ must conv. at 0

Book: $\int_0^x f(t)\,dt$
gives one antideriv. of $f(x)$.

gives AD w/ $C=0$ if $\int$ Power Σ

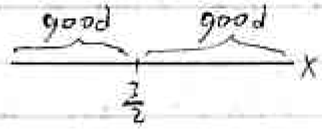
In Ⓐ we found power Σ rep
In Ⓑ we D_x
Now, in Ⓒ, we $\int$

Ex (#4) $f(x) = \frac{1}{3-2x}$

Math 150 $\int \frac{1}{3-2x}\,dx \qquad u = 3-2x$

$\vdots$

$$= -\tfrac{1}{2}\ln|3-2x| + C$$



$\int_a^b \frac{1}{3-2x}\,dx$
OK if $\frac{3}{2}$ not in $[a,b]$
Can use FTC directly.

Use power Σ rep. in Ⓐ

$$\int \frac{1}{3-2x}\,dx = \int \sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}} x^n\,dx \quad , \ |x| < \tfrac{3}{2} \ (R = \tfrac{3}{2})$$

$$= \sum_{n=0}^{\infty} \int \frac{2^n}{3^{n+1}} x^n\,dx$$

indep. of x

$$= \sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}} \cdot \left[\frac{x^{n+1}}{n+1}\right] + C$$

$$= \sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}(n+1)} x^{n+1} + C$$

(represent same family of funcs. (corresp. to ∥ graphs) on I)

$$\left(= -\tfrac{1}{2}\ln|3-2x| + \hat{C} \text{ on its } I\right)$$

Stewart 11.9.37
$\sum \frac{x^n}{n^2}$ Ⓒ at endpts. but f' Ⓓ

Larson 612
f' Ⓒ at $\pm d$ $\Rightarrow$ so does f

$D_x, \int$ (geom) $\Rightarrow$ not geom

Up to 7
Up to 9 - if assign (no)

$$\left.\begin{array}{l} \text{Again, } R = \frac{3}{2} \\ \text{Turns out } I = [-\frac{3}{2}, \frac{3}{2}) \end{array}\right\} \text{for new } \sum$$

had to ✓ endpts. separately

(When you D_x or $\int$ a Power $\sum$, $\Rightarrow$ same R, but the behavior at the endpts. of I may change.)

In M150 we used Newton's Method to approx. π.
4.8.25:
$f(x) = \sin x$
$x_{n+1} = x_n - \tan x_n$
but how compute?

Now, we only know how to expand a func. using geom. template.
11.8 - we'll see a direct expansion

① Find a power $\sum$ rep. for $\tan^{-1} x$, and use it to approx. π to 2 decimal places.

(See Ex 5 on p. 576 - we're simplifying this.)

$$f(x) = \tan^{-1} x$$

How can we use the Geometric Template?
(Another method in 11.8)

$$f'(x) = \frac{1}{1+x^2}$$
$$= \frac{1}{1-(-x^2)}$$

Use: $\frac{1}{1-A} = \sum_{n=0}^{\infty} A^n$, $|A| < 1$

$$= \sum_{n=0}^{\infty} (-x^2)^n, \quad |-x^2| < 1$$
$$|x| < 1$$
$$R = 1$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

$f(x) = \int f'(x)\,dx$ (actually, one member of this family of antiderivs.)

$\tan^{-1}x = \int \sum_{n=0}^{\infty} (-1)^n x^{2n}\,dx$

$\tan^{-1}x = \sum_{n=0}^{\infty} (-1)^n \int x^{2n}\,dx$

Stewart does +C and C+

$\tan^{-1}x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)} + C$ (Some write $C + \sum \cdots$)

(Can pull out, if you want)

Solve for C

If had $\sum \cdots (x-3)^{2n+1}$ use $x=3$.

$x = 0 \Rightarrow$ (we know sum of $\sum$.)

$\tan^{-1}(0) = 0 + C$

$0 = C$

$C = 0$

Stewart 758: Called Gregory's Series after James Gregory (Scottish, 1638-1675) who discovered it in 1671. Madhava (Indian) knew of it in 1400!! } History

$$\Rightarrow \boxed{\begin{aligned}\tan^{-1}x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \\ &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\end{aligned}}$$

$\{2n+1\} \Rightarrow$ odd #s

$\{2n\} \Rightarrow$ even #s

$R=1$ for this $\sum$, also.

Find I for this $\sum$.

D ? C ? D

-1 1

If $x=-1$: $\sum_{n=0}^{\infty} (-1)^n \frac{(-1)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^{3n+1} \frac{1}{2n+1}$ $\to 0$

Alt. $\sum$

C by AST

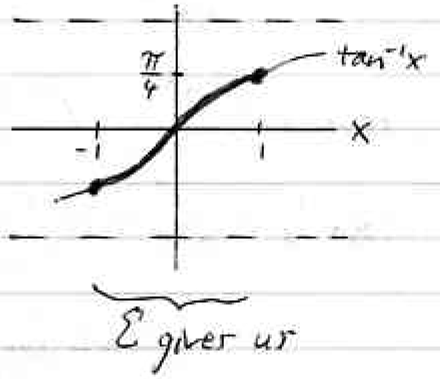
If $x=1$: $\sum_{n=0}^{\infty} (-1)^n \frac{(1)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}$ $\to 0$

Alt. $\sum$

C by AST

$\boxed{\text{New } I = [-1, 1]}$

For any x in I, sum of $\Sigma = \tan^{-1}x$.



<u>Note</u> odd func., so
odd-only exponents in Σ
not a coincidence

Remember, $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$

<u>Plug in $x = 1$</u>

$\pi \approx 3.14159265358979\ldots$
Now, π nice!

$$\tan^{-1}(1) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots,$$

$$\frac{\pi}{4} =$$

$$\boxed{\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \ldots} \quad \left(\pi = \sum_{n=0}^{\infty} (-1)^n \frac{4}{2n+1}\right)$$

Irrational! — All terms rational! Nice!

(The sum of a <u>finite</u> # of rational #s must be rational.)

Stewart 758: Leibniz formula for π. He plugged in $x=1$; independently discovered $\tan^{-1}$ series in 1674, three years after Gregory. The issue of plugging in $x=1$ is dealt with in a "Note." } History

Approx. π to 2 dec. places

Turns out you need 400 terms...

Need $|\text{error}| < 0.005$ $\left(= \frac{1}{200}\right)$

$$\pi = \underbrace{4 - \frac{4}{3} + \dots - \frac{4}{799}}_{400 \text{ terms}} + \frac{4}{801} - \dots$$

S_{400}? We start w/0.

$$\left(|\text{error}| \le \left|\frac{4}{801}\right| \approx 0.00499 < 0.005\right)$$

3.13909

$$\boxed{\pi \approx 3.14}$$

Amounts not shrinking fast enough [illegible]

Beckmann 140

Like Spears - good-looking but mostly useless

Note: Like the alt. harmonic Σ, this Σ converges slooowly! Need $\approx 10^{50}$ terms for 100-digit accuracy!

Larson 8.9, 43, 44
Edwards 572-3
$\tan^{-1}\left(\frac{1}{5}\right) \approx$ geom.

In 1706, John Machin used the following to approx. π to 100 places:

$$\frac{\pi}{4} = 4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right)$$

(CAN SKIP!!)

Highly Technical

Note Stewart (5th ed, 758) observes:
"We have shown that Gregory's ($\tan^{-1}x$) series is valid when $-1<x<1$, but it turns out (although it isn't easy to prove) that it is also valid when $x=\pm 1$."

Alternate pf. for ln 2: Ross 178 $\ln 2 = \ln(1+x)$ $R_n(1)\to 0$

Ross 147 Hard to prove

The key is Abel's Theorem:
Let's say $R=d$.
If Σ conv. at d, $-d$, then Σ is continuous at d, $-d$.

Here Since Gregory's $\tan^{-1}x$ Σ conv. at $x=1$, then, by Abel's Thm., the Σ is cont. at 1. (one-sided)
$\tan^{-1}x$ is also cont. at 1, so the Σ evaluated at $x=1$ must agree w/ $\tan^{-1}(1)$.

It's OK that $\underbrace{\frac{1}{1+x^2}}_{\text{our } f'(x)\ \Sigma}$ only had $I=(-1,1)$.

Stewart 768: Newton often ∫'s this way.

Ⓔ Exs (Newton often ∫ed this way!)

22: Σ rep. for $x^4 \arctan(x^4)$
Parts? No...
If you can do this in 1 min, you don't know what you're doing.
{2n+1} sweeps out over todd int.

Ex $\int x \tan^{-1}(x^3)\,dx$, $\underbrace{|x|<1}_{\text{Limited!}}$ (Can't do $\int_2^3 \cdots$)

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad \text{from } Ⓓ$$

$$\tan^{-1}(x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n+1}}{2n+1}, \quad |x^3| \le 1,\ |x| \le 1 \quad (R=1)$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+3}}{2n+1}$$

What's the one change we make?

$$x \tan^{-1}(x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+4}}{2n+1}$$

Now, we can ∫!! Don't forget +C!!

$$\int x \tan^{-1}(x^3)\,dx = \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+5}}{(2n+1)(6n+5)} + C \quad (R=1)$$

We can't do $\int_0^{\infty}$ using Σ

Ex $\int_0^{0.9} x \tan^{-1}(x^3)\,dx \overset{FTC}{=} \left[\sum \quad \right]_0^{0.9}$

≈ 0.1101

≈ 0.1086

$$= \left[\frac{x^5}{5} - \frac{x^{11}}{33} + \ldots \right]_0^{0.9}$$

$$= \frac{(0.9)^5}{5} - \frac{(0.9)^{11}}{33} + \ldots - \cancel{0}$$

satisfies hypoths. of AST

|error| ≤ |1st neglected term|

How can we rewrite this so we can expand? Parts?! No as a Σ? Swok doesn't mention →

How can we rewrite so we can ⇒ expand as a Σ?

Ex $\int \overbrace{\frac{\tan^{-1} x}{1+x^5}}\,dx = \int (\tan^{-1} x)\left(\frac{1}{1+x^5}\right) dx$

(Book doesn't do!)

$$= \int \underbrace{\left(x - \frac{x^3}{3} \ldots\right)\left(1 - x^5 + x^{10} \ldots\right)}_{\text{Distribute (find all terms of degree <20, say?)}} dx$$

or Long ÷ $1+x^5 \overline{\smash{)}\, x - \frac{x^3}{3} \ldots}$ (Risky) (Theory not as clean)

Still up to 7/9

If $\int \underset{\sim}{\Sigma} \sim \underset{\sim}{\Sigma} \sim dx$ if same R ⇒ same R for resulting Σ

∫ term-by-term, + C

(R = 1) ⇐ Don't have to find here.

Grid of 8 funcs:
So far, $\frac{1}{1-x}$, $\tan^{-1}x$

Larson 618:
Modern calc devices use E. (See L11-38)

Weird pf in 11.7
formula in 11.8
Newton:
$1+x = e^{\ln(1+x)} = \sum a_n (\quad)^n$
match coeffs.

need if $x<0$

Ⓕ e^x

Turns out (I'll convince you in 11.8)

$$\boxed{\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \\ &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots \\ &\text{for all real } x \quad (R=\infty) \ (n! \text{ in den. "helps"}) \\ &I = (-\infty, \infty) \end{aligned}}$$

calculator just needs $+, -, \cdot, \div$, constr. to approx. $e^{2.91}$, say

Note: $e^0 = 1$

Shifting over 1 position

$$D_x(e^x) = 1 + x + \frac{x^2}{2!} + \dots$$
$$= e^x$$

where's the 1?

$$\int e^x\,dx = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + C$$

spits out 1

$$= e^x + C$$

$C = 1 + \hat{C}$ confused them!

If $x = 1$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$
$$= 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} + \dots$$

Sum of recips of facs

up to 17

Note: Although it's easy to compute 2!, 3!, we like to indicate the pattern.

before sinh x "+" more convenient

(G) cosh x

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \text{(by def'n: p. 440)}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$
$$e^{-x} = 1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \dots$$
$$= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

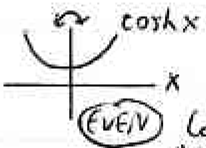
$$e^x + e^{-x} = 2 + 2\cdot\frac{x^2}{2!} + 2\cdot\frac{x^4}{4!} \dots$$

$$\boxed{\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} \dots}$$

{2n} ⇒ even #s

$$\boxed{\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \quad \text{for all real } x}$$

Dictionary of Math: catenary: $a \cosh\left(\frac{x}{a}\right)$ a = y-int



"catenary" hanging cable suspension bridge

EVEN Look at exponents!! "1" OK

ODD

(H) sinh x

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

Can prove using def'ns

Also, $D_x(\cosh x) = \sinh x$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$D_x\left(\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}\right) = \sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!}$$

$$\boxed{D_x(\cosh x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots}$$

{2n+1} ⇒ odd #s

$$\boxed{\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \quad \text{for all real } x}$$

Why can't we have a Σ for $\ln x$ centered at 0? Instead, $\ln(1+x)$.

Ex 3 p.575 $\ln(1+x) = \int_0^x \frac{1}{1+t}dt$

(I) $\ln(1+x)$

I want a Σ. How can I start? How can I use the geom template?

$$f(x) = \ln(1+x)$$

$$f'(x) = \frac{1}{1+x}$$

6 of 8 so far. Last - $\sin x$, $\cos x$. Geom. template not helpful. Not related directly to e^x.

$$= \frac{1}{1-(-x)}$$

$$= \sum_{n=0}^{\infty} (-x)^n, \quad |-x|<1$$

$$= \sum_{n=0}^{\infty} (-1)^n x^n, \quad |x|<1$$

$$R=1$$

We're sacrificing precision for convenience.

$$\Rightarrow f(x) = \int \sum_{n=0}^{\infty} (-1)^n x^n \, dx \quad \text{(one member)}$$

$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} + C$$

$$x=0 \Rightarrow$$
$$\ln 1 = 0 + C$$
$$0 = C$$
$$C = 0$$

$$\boxed{\begin{aligned}\ln(1+x) &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} \\ &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\end{aligned}}$$

or $\sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^k}{k}$

Sub: $k=n+1$
$\Rightarrow n=k-1$
(Common trick in 255 Diff Eqs.)

for what x?

(Note: Σ is altern. for $x>0$, not for $x<0$. Plug in a value for x before you think about altern. Σ.)

Find I

$R = 1$, again

D ? C ? D
$\longleftarrow\!\!|\!\!-\!\!-\!\!|\!\!-\!\!-\!\!|\!\!\longrightarrow x$
-1 0 -1

If $x = -1$

Is this an alt. Σ?

$$\sum_{n=0}^{\infty} (-1)^n \frac{(-1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \underbrace{(-1)^{\overset{\text{always odd}}{2n+1}}}_{=-1} \frac{1}{n+1}$$

$$= -\sum_{n=0}^{\infty} \frac{1}{n+1} \text{ or } -\sum_{k=1}^{\infty} \frac{1}{k} \text{ (-Harmonic } \Sigma\text{!!)}$$

$(k = n+1)$

D

L11-29f: Abel's Thm: Σ C at R $\Rightarrow$ cont. & discont. $\Rightarrow$ Σ D

Note: $\ln(1 + (-1)) = \ln 0$
undefined!

If $x = 1$

Is THIS an alt. Σ?

$$\sum_{n=0}^{\infty} (-1)^n \frac{(1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \underbrace{(-1)^n}_{\text{alt.}} \underbrace{\frac{1}{n+1}}_{\ldots \to 0}$$

C by AST

L11-29f Abel's Thm: Σ C at R $\Rightarrow$ cont. If $\ln(1+x)$ cont. there $\Rightarrow$ they agree
(Conv. very slooowly
See Ross 178 for Taylor rem. approach

Note: Plug in $x = 1 \Rightarrow \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ becomes

$$\ln(1+1) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$$

$$\boxed{\underset{\approx .693}{\ln 2} = \text{sum of alt. harm. } \Sigma}$$

Converges slooowly, though!!

$$\boxed{I = (-1, 1]}$$

$\sqrt{2}$: Newton's method

binomial Σ for $(1+x)^k$, $|x| < 1$