7.6 LAWS OF GROWTH AND DECAY and
19.1 SEPARABLE DIFFERENTIAL EQS.

Exs Pop. growth, Cont. Compounded Interest
Radioactive decay

If the rate of change of f (differentiable) with respect to x is proportional to the value of f, what can f be?

$$y = f(x)$$

Solve: $y' = ky$

k is like a constant of proportionality.

Note that $y=0$ is a sol'n.
For now, assume $y>0$ "where we care."

If $k > 0 \Rightarrow y' > 0 \Rightarrow$ growth in y
$\quad\; <\qquad\quad <\qquad$ decay

$$\frac{dy}{dx} = ky$$

Separation of variables

$$\frac{dy}{y} = k\,dx \quad \leftarrow \text{Differential form}$$

$$\underbrace{\frac{dy}{y}}_{\left(\substack{\text{Stuff}\\ \text{in } y,dy}\right)} = \underbrace{k\,dx}_{\left(\substack{\text{Stuff}\\ \text{in } x,dx}\right)}$$

We shouldn't say, "We multiplied by dx." Whatever....

$$\int \frac{dy}{y} = \int k\,dx$$

$$\ln|y| = kx + C$$

Can remove | | ($y>0$)

$$e^{\ln y} = e^{kx+C}$$

$$y = e^{kx}\underbrace{e^{C}}_{=D}$$

$$y = De^{kx}$$

or $f(x)$

If $x=0 \Rightarrow f(0) = D\underbrace{e^{k(0)}}_{=1}$

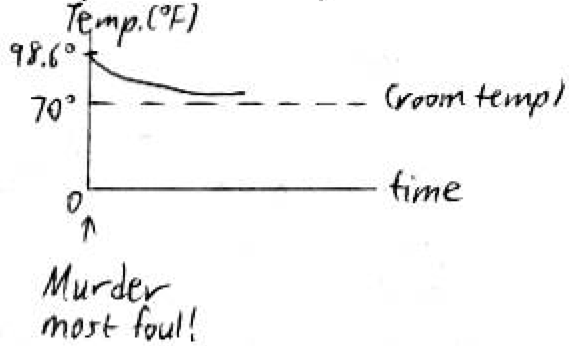
$$\text{“}y_0\text{”} = D$$

$$\boxed{y = y_0 e^{kx}}$$

<u>Ex 3</u> (pp. 418-9) Newton's Law of Cooling

Dead professor problem

98.2°?
40°?



Ex Solve $\cos x \cot x \frac{dy}{dx} + 7y = 0$

Assume $y \neq 0$ (Note that $y=0$ is a sol'n.)
$\cos x, \cot x \neq 0$, are defined
"where we care"

Differential form:

$$\cos x \cot x \, dy + 7y \, dx = 0$$

$$\cos x \cot x \, dy = -7y \, dx$$

$$\int \frac{dy}{y} = \int \frac{-7}{\cos x \cot x} dx$$

$$\ln|y| = \int -7 \sec x \tan x \, dx$$

$$\ln|y| = -7\sec x + C$$

$$e^{\ln|y|} = e^{-7\sec x + C}$$

$$|y| = e^{-7\sec x} e^{C}$$

$$y = \pm e^{C} e^{-7\sec x}$$

$$y = De^{-7\sec x}$$

Note: $D \neq 0$, though
$D = 0 \Rightarrow y = 0$ (sol'n)

Recommended: 19.1, #11

(6.6) WORK

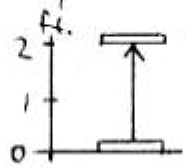
If constant force F $\longrightarrow$ (object) $\longrightarrow$ moves a distance of d units

Work done $W = Fd$

F ↑ force, d ↑ distance

Ex A 50-pound rod is lifted 2 feet.

50 pounds is the magnitude of the force required to lift the rod.

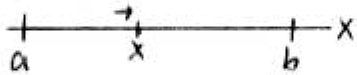


$W = Fd$
$= (50 \text{ pounds})(2 \text{ feet})$
$= 100$ foot-pounds

Metric: 1 Newton-meter (N-m), or
1 joule (J)

≈ 0.74 foot-pounds

Ex F: variable force (continuous)

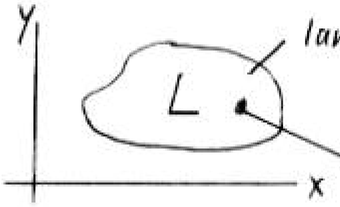


Work done in moving an object from $x=a$ to $x=b$ along —— x =

$W = \int_a^b F(x)\,dx$

(6.7) MOMENTS and CENTERS OF MASS

Larson 458, 6ed



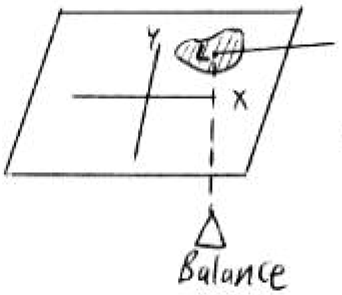
lamina (flat plate) consists of point masses. Really, there has to be thickness.

the density at (x,y) is $\rho(x,y)$.
Here, it's mass per unit area, really, volume normally

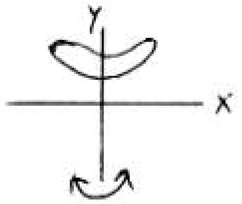
For now, assume $\rho(x,y)=1$ throughout L.

constant, so <u>homogeneous</u>

Then, mass = Area

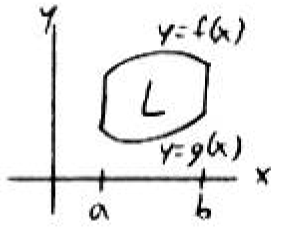


The centroid of the region $(\bar{x},\bar{y})$ is the center of mass of L.



$\bar{x}=0$ by symmetry

I won't test you on this page, but take a look!



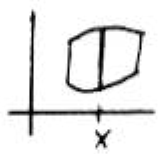
Find the centroid $(\bar{x},\bar{y})$ of L.

$m =$ mass of L
$=$ area
$= \int_a^b [\underset{\text{top}}{f(x)} - \underset{\text{bottom}}{g(x)}]\,dx$

$\bar{x} = \frac{M_y}{m}$ ← Moment of L about the y-axis } like an average

"adding" there: $x \cdot$ weight at x

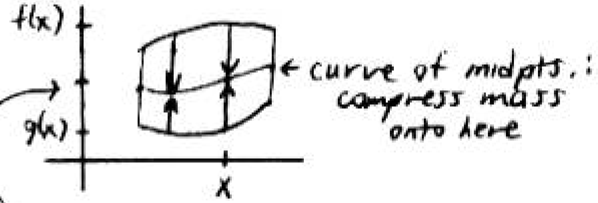
where $M_y = \int_a^b x \underbrace{[f(x)-g(x)]}_{\text{weight at } x}\,dx$



$\bar{y} = \frac{M_x}{m}$ ← Moment of L about the x-axis

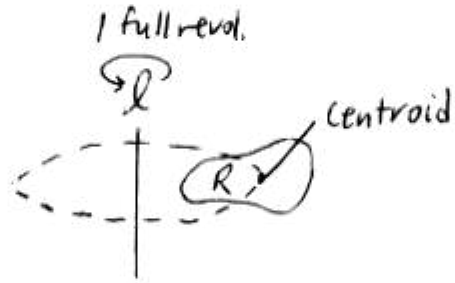
where $M_x = \underbrace{\int_a^b \underbrace{\tfrac{1}{2}[f(x)+g(x)]}_{\text{(☆) } y\text{-coord. on curve of midpts.}} \cdot \underbrace{[f(x)-g(x)]}_{\text{weight at } x}\,dx}_{\text{weighting } y\text{-coords.}}$

Why ☆?



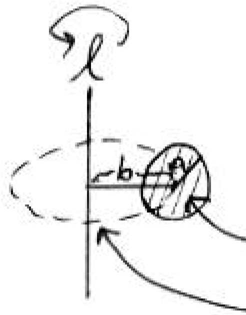
average $= \frac{1}{2}(f(x)+g(x))$

Theorem of Pappus (Cool!)



Volume of resulting solid
$= (\text{Area of } R)\left(\begin{array}{c}\text{distance}\\ \text{traveled by}\\ \text{centroid}\end{array}\right)$

Ex (Donut/Torus)



$$V = (\pi a^2)(2\pi b) = 2\pi^2 a^2 b$$

6.7: MOMENTS and CENTERS OF MASS

(A) One-Dimensional Setting

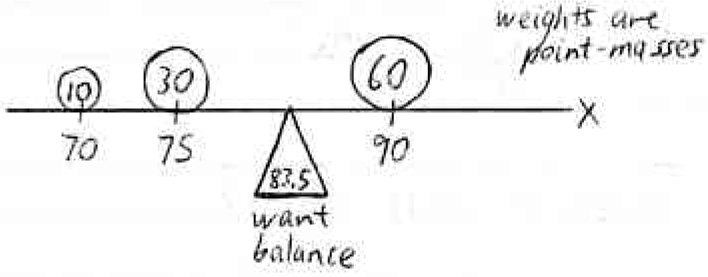
relevant to your every day lives

Ex

Test #k	Weight (% of class grade) (m_k)	Your score (x_k)
Test 1	10	70
Test 2	30	75
Test 3	60	90
(n=3 tests)	$\sum_{k=1}^{3} m_k = 100$	

quiz
mid
final
no HW-lucky!
what do you want to know?

Find your weighted % for the class.



not boulders

mean

$\bar{X}$ = center of mass

$$= \frac{\sum_{k=1}^{3} m_k x_k}{\sum_{k=1}^{3} m_k} \quad \begin{array}{l} \leftarrow \text{moment "}M_0\text{" about the origin} \\ \leftarrow \text{"}m\text{", the total mass} \end{array}$$

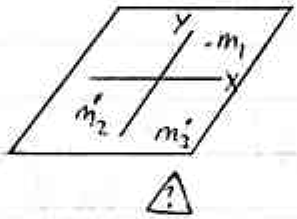
The quiz was worth 10%.

$$= \frac{(10)(70) + (30)(75) + (60)(90)}{100} \rightarrow \text{Press } \boxed{=}$$

$$= 83.5$$

(B) Two-Dimensional Setting

m_1 is at (x_1, y_1)

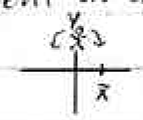


For $k = 1, 2, \ldots, n$:
The point-mass m_k is at (x_k, y_k).

Let $m = \sum_{k=1}^{n} m_k$
= total mass

You're standing somewhere on the y-axis and you're looking around for $\bar{x}$.

$$\bar{x} = \frac{\sum_{k=1}^{n} m_k x_k}{m} \leftarrow \text{"}M_y\text{", the moment about y-axis}$$



$$\bar{y} = \frac{\sum_{k=1}^{n} m_k y_k}{m} \leftarrow \text{"}M_x\text{"}$$

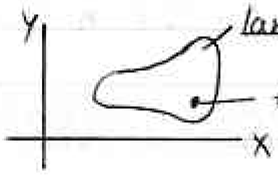
Center of mass: $(\bar{x}, \bar{y})$

(C) The Centroid of a Region

p. 426

Calc III

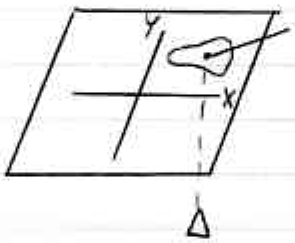
(in III, use δ)



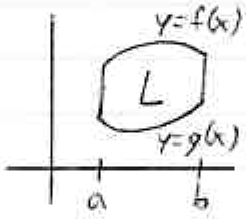
Calc I

Assume $\rho(x,y) = 1$ throughout the lamina.
homogeneous (constant density)

Then, $\underset{\text{(total mass)}}{m} = \underset{\text{(area)}}{A}$.



$(\bar{x},\bar{y})$ is the center of mass of the lamina, or the <u>centroid</u> of the region



Find the centroid $(\bar{x},\bar{y})$ of L.

$m =$ mass of L

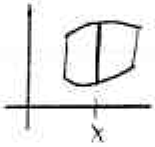
$\quad =$ area

$$= \int_a^b [\underbrace{f(x)}_{top} - \underbrace{g(x)}_{bottom}]\,dx$$

$$\bar{x} = \frac{M_y}{m}$$

adding there x · weight at x

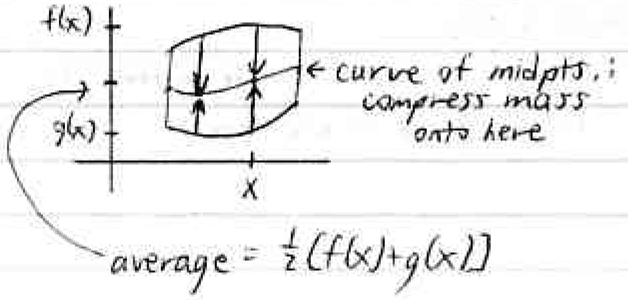
$$\text{where } M_y = \int_a^b x \underbrace{[f(x)-g(x)]}_{\text{weight at } x}\,dx$$



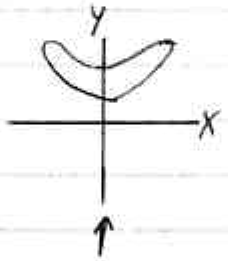
$$\bar{y} = \frac{M_x}{m}$$

$$\text{where } M_x = \int_a^b \underbrace{\tfrac{1}{2}[f(x)+g(x)]}_{\text{y-coord. on curve of midpts.}} \cdot \underbrace{[f(x)-g(x)]}_{\text{weight at } x}\,dx$$

weighting y-coords.



Read Exs. 3-6



$\bar{x}=0$ by symmetry