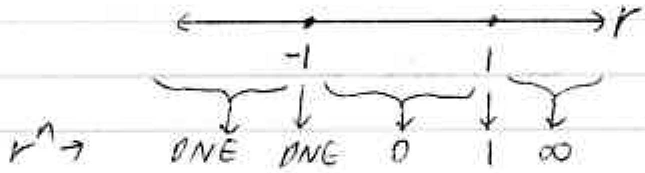
11.1-11.5: REVIEW

11.1: SEQUENCES $\{a_n\}$

Geom. $\{ar^{n-1}\}$ $a \neq 0$

Can say Conv/Div.
D D C C D



$(-1)^{n[\pm 1]}$: Sign alternators

$\lim\limits_{n\to\infty} a_n$ Recall: Squeeze, L'H; Consider $\lim\limits_{x\to\infty} \underbrace{f(x)}_{\text{interpolating func.}}$

$(a_n \to 0) \Leftrightarrow (|a_n| \to 0)$

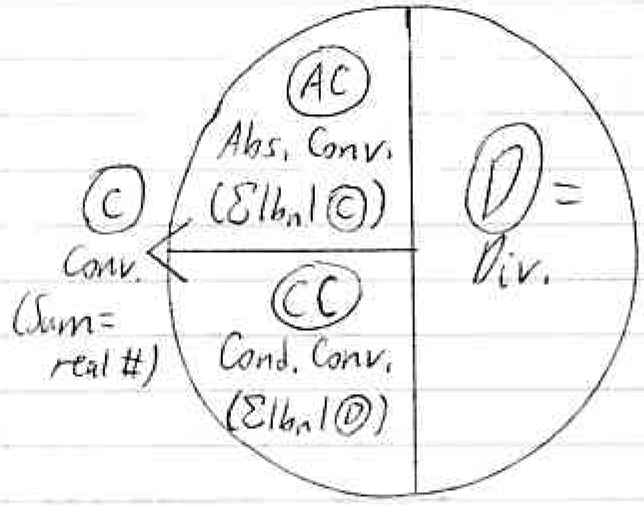
$(1+\frac{1}{n})^n \to e$

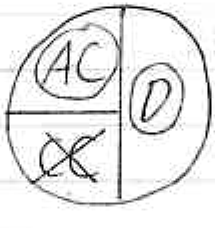
11.2-11.5: SERIES (Σ)

Chart p. 565

$\sum b_n$

"+" Term Σ
($\Sigma |b_n|$ same as Σb_n)





C, D not affected by:

(a) Inserting / dropping / changing a finite # of terms.
("Eventually": for all $n \geq$ some N)

(b) $\cdot, \div$ by a non-0 #

Linear Combos

e.g., $\Sigma(7a_n + b_n - 9c_n)$

All Σs C $\Rightarrow$ C
1 Σ D $\Rightarrow$ D
≥ 2 Σ D $\Rightarrow$?

Tests for C/D (State test when you use it.)

1 Famous Families of Σ

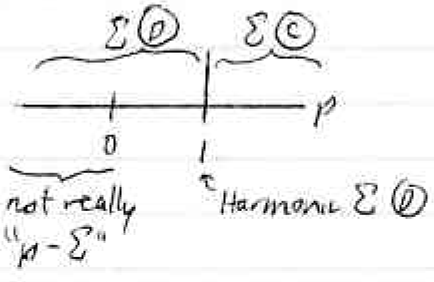
1a Geom. $\sum ar^{n-1}$ $(a \neq 0)$

C $\Leftrightarrow |r| < 1$
else D

Technically, $p > 0$ for a "p-Series"

1b p-Series $\sum \frac{1}{n^p}$

C $\Leftrightarrow p > 1$
else D



(2) n^{th}-Term Test for (D)

If $\sum a_n \not\to 0 \Rightarrow$ (D)

(3) Comparison Tests (for "+" Term Σ)

Given: $\sum a_n$
Find: $\sum b_n$ (Comparison Σ)
Usu. geom., p-series
Take dominant terms from a_n
For LCT, make constant factors $= 1$
Good if a_n algebraic (rational, though roots OK); compare w/ p-Σ
To show $\sum a_n$ (C)/(D), pick $\sum b_n$ that (C)/(D)

(3a) BCT

Ex $\frac{1}{n^2+1} \leq \frac{1}{n^2}$ — If $\geq \Rightarrow$ Use LCT

Big Σ (C)

$\Rightarrow$ Little Σ (C)

Ex $\frac{1}{n} \leq \frac{1}{n-1}$

Little Σ (D) $\Rightarrow$ Big Σ (D)

(3b) LCT

If $\lim\limits_{n \to \infty} \frac{a_n}{b_n} =$ non-0 real # ("not special")

$\Rightarrow \sum a_n, \sum b_n$ both (C)
or both (D)

(4ab) Ratio, Root Tests

L: $\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\|$	$\lim_{n\to\infty} \sqrt[n]{\|a_n\|}$
Good if $n!$, products, 2^n, ...	Good if $(\ )^n$
	$n^{1/n} \to 1$

Bad if a_n algebraic, since $\Rightarrow L = 1$. Use BCT, LCT?

Both: $L < 1 \Rightarrow$ AC
$L = 1$, "DNE" $\Rightarrow$?
$L > 1$, "∞" $\Rightarrow$ D

(5) $\int$ Test (for "+" Term Σ)

Ex $\sum_{n=3}^{\infty} \frac{1}{n} \Rightarrow$ Consider $\int_3^\infty \frac{1}{x}\,dx$

Need: can $\int$ interpolating func $f(x)$ models $\{a_n\}$
+, cont., ↘ (Show $f'(x) < 0$)

If $\int$ C $\Rightarrow \Sigma$ C
$\int$ D $\Rightarrow \Sigma$ D

Good if $\ln n$ and $\frac{1}{n}$? u-sub

6 Tests for C if Terms may be "−"

6a Alt. Series Test (AST) (for conv. of an alt. Σ)

$$\underbrace{\sum (-1)^{n[\pm 1]} \underset{>0}{a_n}}_{b_n}$$

If 0 alt. Σ ✓
1 $a_n \cdots \searrow$ (Show?)
2 $a_n \to 0$ (Show?)

$\Rightarrow \sum b_n$ C

6b Abs. Conv. Test (ACT)

If I can show $\underbrace{\sum |b_n|}_{\text{"+" Term } \Sigma}$ C

$\Rightarrow \sum b_n$ C, AC (Don't need AST.)

CCT: ≥2 steps?

Also n^{th}-Term Test for D
Ratio, Root Tests for C, D
Alt. Geom. Σ are Geom. Σ.

If we rearrange...

AC $\Sigma \Rightarrow$ new Σ w/ same sum
CC $\Sigma \Rightarrow$ can get any real sum (or no sum)

FINDING SUM, S

① Geom. $\sum_{n=1}^{\infty} ar^{n-1}$

$$S = \frac{\overset{\text{1st term}}{a}}{1-\underset{\text{common ratio}}{r}} \quad \text{if } |r|<1 \ (\Sigma \text{ ©})$$

② Telescoping Σ

$\sum \underset{\text{rational}}{a_n}$

<u>Ex</u> $\sum_{n=1}^{\infty} \frac{5}{(5n+2)(5n+7)}$

Find S_k using pfd, canceling.

$S = \lim_{k\to\infty} S_k$ (exists $\Leftrightarrow$ Σ ©, else, Ⓓ) } True for <u>all</u> Σ

APPROX S by S_N

① $\sum_{n=1}^{\infty} \underset{+,\ \searrow}{a_n}$: $S_N < S < S_N + \overbrace{\int_N^{\infty} \underbrace{f(x)}_{+,\ \text{cont.},\ \searrow} dx}^{\text{upper bound on error}}$

interpolating func. models $\{a_n\}$

If you last added $(+a_N)$, then S_N is an upper bound for S. If last subt. $(-a_N)$, then lower bound.

② Alt. $\sum (-1)^{n(\pm 1)} \underset{\text{always } \to 0}{a_n}$: $\underbrace{|\text{error}|}_{|S-S_N|} \le \underbrace{|\text{1st neglected term}|}_{a_{N+1}}$ "Look ahead"

COOL! (Don't worry for now) $\sum_{n=0}^{\infty} \frac{1}{n!} = e$, Alt. Harm. Σ: $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n} = \ln 2$

Mathematica $\Rightarrow$ $\sum_{1}^{\infty} (-1)^{n-1}\frac{1}{n^2} = \frac{\pi^2}{12}$, $\sum_{1}^{\infty} (-1)^{n-1}\frac{1}{n^4} = \frac{7\pi^4}{720}$

From Fourier Series, Advanced Complex Analysis, Euler; we can find sums of p Σ, p even $\Rightarrow$ $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$ ...