REVIEW 11.6-8

Basic M Series ($\frac{1}{1-x}$, and p. 587)

Know: 1st 4 non-0 terms
M series in $\sum_{n=0}^{\infty}$ form
I; it's $(-\infty, \infty)$ unless otherwise specified (①, ⑦, ⑧) No "!" in denom.

① $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots = \sum_{n=0}^{\infty} x^n$, $I = (-1, 1)$ (Basic Geom. Σ)

$e = \sum_{n=0}^{\infty} \frac{1}{n!}$ = sum of recips of !s

② $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ ($D_x(e^x) = e^x$)

Alt. Σ $\forall x \neq 0$. If $x<0 \Rightarrow - + - +$ after plug-in

③ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ $\downarrow D_x$

④ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

Alt. Σ (for all $x \neq 0$), $\sin x$ is <u>odd</u>, $\cos x$ is <u>even</u>
($\Rightarrow$ makes periodicity possible)

⑤ $\sinh x$ = (same as $\sin x$, except all "+"s) $= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ $\updownarrow D_x$

⑥ $\cosh x$ = (" " $\cos x$ ") $= \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

$\ln 0$ is und., so $\ln x$ has no M series

⑦ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$

or $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

From $\int \frac{1}{1+x}\,dx$, $C=0$

<u>Remember: When</u> $x = 1 \Rightarrow \ln 2 = \underbrace{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots}_{\text{sum of alt. harm. } \Sigma}$

$I = (-1, 1]$
~~ln 0~~ $\ln 2$ ✓

⑧ $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots = \underbrace{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}}_{\text{From } \int \frac{1}{1+x^2}dx,\ C=0}$

Alt. Σ (for all $x \neq 0$), $\tan^{-1} x$ is <u>odd</u>

<u>No "!"s</u>: $\tan^{-1}(1)$ converges <u>slowly</u> to $\frac{\pi}{4}$.

$I = [-1, 1]$

Work out as last resort ①-⑦

Not good for ⑧ →

M Series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ Pattern?

T Series centered at c: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n$

Where valid? $\exists f^{(n)}$ $(n=0,1,2,\dots)$, $R_n \to 0$

<u>Manipulations</u>

Sub

<u>Ex</u> $\sin(x^2)$

<u>Ex</u> $\frac{x^3}{4+x^3} = \underbrace{\frac{x^3}{4}}_{\text{Can} \cdot \text{ in later}} \cdot \underbrace{\frac{1}{1-(-\frac{x^3}{4})}}$

$\Rightarrow$ Geom. Σ, $|-\frac{x^3}{4}| < 1 \leftrightarrow |x| < \sqrt[3]{4}$

$f'(x)$, $\int f(x)\,dx$ have same R as for f, I up to endpoints (if any)

$\Sigma a_n x^n$: $\underbrace{n=0}_{\text{if this term is constant}} \to n=1$; $+C$

<u>(11.6) Power Σ in general</u>

Find I: For what x is "Ratio Test L" < 1? (or Root)

✓ endpts. separately.

	<u>$\Sigma a_n x^n$</u>	<u>$\Sigma a_n (x-c)^n$</u>
$R=0$	© for $\{0\}$	© for $\{c\}$
$R=\infty$	$(-\infty,\infty)$	$(-\infty,\infty)$
$R=d>0$	$\{-d, d\}$	$\{c-d, c+d\}$

<u>Ex</u> Solve $3|x-2| < 1$