CH. 13 REVIEW

(13.1) Plane Curves

Graph C: $x = f(t)$, $y = g(t)$, t in I

May help to get rect. eq.

① ETP

② Directly relate x, y

Ex $\left.\begin{matrix} x = 3t \\ y = 9t^2 \end{matrix}\right\} \Rightarrow y = x^2$

③ Use IDs

Ex $\left.\begin{matrix} x = \cos t \\ y = \sin t \end{matrix}\right\} \Rightarrow x^2 + y^2 = 1$

Restrict graph; Orientation Tools:

① Point-plotting (✓ $t=0$, endpts. of interval; shouldn't be only tool)
② Sign of $\frac{dx}{dt}, \frac{dy}{dt}$
③ ETP, ④ Use IDs, other relationships
④ Restrictions on x, y (Sign, extreme values)
⑤ As $t \nearrow$, does $x \nearrow, \downarrow$? y? When?
⑥ As $t \to \infty$ or $\to -\infty$ ...
⑦ Graph x and/or y against t.
⑧ Templates (Circles, Ellipses, etc.)

13.2

~~tan~~ ~~normal~~ $\quad y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ Hor., vert. tan lines

$y'' = \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$ ← D_t whole thing

$\sim \quad L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

13.3 PCs

Graph r or $r^2 = f(\theta)$

1. θ
2. $r < 0$
3. When does $r \to 0$?
4. $r = 0$? (Pole)

Basic shapes

○, limaçons, ω, ∞, roses

Polar Eq. ⇔ Rect. Eq.

$r^2 = x^2 + y^2$ $\qquad x = r\cos\theta$

$\tan\theta = \frac{y}{x}$ Watch Q! $\qquad y = r\sin\theta$

Tangent lines

$$m = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)}$$

13.4 Ss

$A = \int_a^b \frac{1}{2} r^2\, d\theta$

Sym., ⊗, r_{out} vs. r_{in}, "−" areas

$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$