Kant: Logic is a science of the necessary laws of thought.
Logic is the anatomy of thought. Locke
Logic is the art of going wrong with confidence. Joseph Krutch

1.1: LOGIC

We'll need for 3.1.

Gateway to precise thinking

A <u>proposition</u> is a statement that is either true (T) or false (F).

$\underbrace{\text{true (T) or false (F)}}_{\text{truth values}}$

Not tested

<u>Ex</u>

<u>Are props.</u>	<u>Are not props.</u>
Al Gore is a Dem. (T)	Whazzup?
G.W. Bush is a Dem. (F)	Just do it.
The sky is blue. (<u>but</u> depends on context, opinion)	
$1+1=2$. (T)	
$1+1=3$. (F)	
$2x=10$ if $x=5$. (T)	$2x=10$. [assume we know nothing about x]

right, Texas

In CS, 0 = false, 1 = true.
In <u>fuzzy logic</u> (AI), a proposition can have a truth value between 0 and 1.

Jigsaw
Quantum mech
bits/fits (Kosko 24)

<u>Ex</u> Ken is tall. (0.3)

<u>Overhead</u>

Let p, q, r, etc. denote propositions.
We use <u>logical operators</u> to build <u>compound propositions</u>.

① $\neg p$ means $\begin{cases} \text{not } p \\ \text{the negation of } p \\ \text{it is not the case that } p \end{cases}$

<u>Truth table for $\neg p$</u>

p	$\neg p$
T	F
F	T

If p is a $\frac{T}{F}$ prop., then $\neg p$ must be a $\frac{F}{T}$ prop.

<u>Ex</u> p: $1+1=2$ (T)
$\neg p$: $1+1 \neq 2$ (F)

<u>Ex</u> a, b are real #s
p: $a > b$
$\neg p$: it is not the case that $a > b$
(i.e., $a \leq b$)

one is T; the other is F

Note: $\neg$ is a <u>unary operator</u>.
<u>Connectives</u> are operators that combine 2 or more props.

② $p \wedge q$ means $\begin{cases} p \text{ and } q \\ \text{the conjunction of } p \text{ and } q \end{cases}$

Truth table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

all possibs

$p \wedge q$ is a T prop. $\xleftrightarrow{\text{exactly when}}$
p and q are both T.

Your grandma wears army boots.

Ex p: Gore will win. (?)
q: $1+1=3$. (F)
$p \wedge q$: Gore will win and $1+1=3$. (F, regardless)

③ $p \vee q$ means $\begin{cases} p \text{ or } q \text{ (inclusive or)} \\ \text{the disjunction of } p \text{ and } q \end{cases}$

Truth table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$\left.\begin{matrix} T \\ T \\ T \end{matrix}\right\}$ one or both of p, q T

$p \vee q$ is a F prop. $\longleftrightarrow$
p and q are both F

④ $p \oplus q$ means $\begin{cases} \text{exclusive or of } p \text{ and } q \\ p \text{ xor } q \end{cases}$

Truth table

p	q	$p \oplus q$
T	T	F ← different from $p \vee q$
T	F	T
F	T	T
F	F	F

for purposes of circuit design, it's interesting that $p \oplus q$ is **F** $\leftrightarrow p, q$ have same truth value

$p \oplus q$ is a T prop. $\longleftrightarrow$
exactly one of p or q is T (but not both!)
("soup or salad" idea)

Ex Let's say

p: You have soup. (T)

q: You have salad. (T)

Then, $p \oplus q$: You have soup xor salad. (F)

but $p \vee q$ (T)

Note: In math, "or" is an inclusive or (3), unless otherwise specified.

(5) The implication $p \to q$

$$\underset{\substack{\text{hypothesis}\\ \text{antecedent}\\ \text{premise}}}{p} \to \underset{\substack{\text{conclusion}\\ \text{consequence}}}{q}$$

SNL sketch

Ex d: Pat is a Daddy.

m: Pat is a man.

There are many ways to say $d \to m$

if d, [then] m	m if d
d implies m	m whenever d
d only if m	m is necessary for d
d is sufficient for m	

($p \to q$ is a F prop) $\Longleftrightarrow$ (p is T and q is F)
"Breaking a contract"

Truth table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

} If the hypothesis (p) is F, the prop. $p \to q$ is T "by default".

Ex p: $1+1=3$ (F)
q: Gore will win (?)
Then, $p \to q$: If $\underbrace{1+1=3}_{F}$, then $\underbrace{\text{Gore will win}}_{?}$ (T "by default")

Key: q is T
Math produce

Ex If $\underbrace{1+1=2}_{T}$, then $\underbrace{\text{a lemon is yellow}}_{T \text{ (all we need)}}$. (T)

Why does this sound weird?
no causal relationship

p and q need not be related! (Stats: Correlation does not imply causation. elev.)

Ex If $\underbrace{\text{Pat is a Daddy}}_{p \text{ (or } d)}$, then $\underbrace{\text{Pat is a man}}_{q \text{ (or } m)}$.

Assume Pat is human.

stronger than

This prop. must be T, because [realistically] there's no way p is T and q is F.

Math Ex If an integer ends in a "3", it's an odd #.

Same idea!

FLT, Fermat $x^3+y^3=z^3$, etc. has no positive integer sol.

Math

Think of an "if-then" statement (prop) as a conjecture (unproven hunch)
If the prop is T, we have a theorem we can put in books.
If the prop. is F, no grant $!! Junk!

5 More

The converse of $p \to q$ is $q \to p$. ($p \to q$ and $q \to p$: not equivalent)

The contrapositive of $p \to q$ is $\neg q \to \neg p$.

HW 1.2 #16

(depending on ~~whether~~ the truth values of p, q)

These are "logically equivalent" (Sec 1.2): both are T or both are F.

Ex Write the converse of
"Pat is a man whenever Pat is a Daddy."

Rewrite in "if-then" form:
"If $\underbrace{\text{Pat is a Daddy}}_{p}$, then $\underbrace{\text{Pat is a man}}_{q}$."

The converse is:
"If $\underbrace{\text{Pat is a man}}_{q}$, then $\underbrace{\text{Pat is a Daddy}}_{p}$."

This new prop. is fundamentally different from the old one!

Ex The contrapositive is:
"If $\neg q$, then $\neg p$"
"If Pat is not a man, then Pat is not a Daddy."

This is fundamentally the same as the old prop.!

⑥ The biconditional $p \leftrightarrow q$

means $\begin{cases} p \text{ if and only if } q \\ p \text{ is necessary and sufficient for } q \\ \text{if } p \text{ then } q\text{, and } \underbrace{\text{if } q \text{ then } p}_{\text{"conversely"}} \end{cases}$

($p \leftrightarrow q$ is T) $\Longleftrightarrow$ (p, q are both T or both F)

Truth table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

We tend to look at the table from right to left.

Ex If we know this is a true prop.:
"$\underbrace{\text{Pat is a man}}$ $\leftrightarrow$ $\underbrace{\text{Pat has an Adam's apple}}$," then both are T, or both are F

HW Tips (Relevant: pp. 1-8; ignore Boolean searches, bit strings)

Ex p: I am young.
q: I am stupid.
r: I can attend the movie.

Express $(p \vee q) \to \neg r$ as an English sentence.

Order of ops: ( ) 1st, $\neg$ next, $\vee / \wedge$, $\to / \leftrightarrow$

"If I am young or stupid, then I cannot attend the movie."

Ex (23d) Construct a truth table for $(p \to q) \wedge (\neg p \to q)$

p	q	$(p \to q)$	$\neg p$	q	$(\neg p \to q)$	$\wedge$
T	T	T	F	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	F	T	T	F	F	F

Different approaches ok!

Concise answer:

p	q	$(p \to q) \wedge (\neg p \to q)$
T	T	T
T	F	F
F	T	T
F	F	F

19a) Converse:

If I will ski tomorrow, then it snows today.

Let's say both are T.

Then, this implication is T.

In conversation, a cause-and-effect relationship is implied.

Our sense of logic does <u>not</u> focus on content or cause-and-effect.