1.2: PROPOSITIONAL EQUIVALENCES

A <u>tautology</u> is a compound proposition that is <u>always T</u>, regardless of the truth values (T/Fs) of its constituent props.

<u>Ex</u> $p \vee \neg p$ (Hamlet is or is not.)

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

→ tautology!

A <u>contradiction</u> is... <u>always F</u>....

<u>Ex</u> $p \wedge \neg p$ (p, $\neg p$ can't both be T.)

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

→ contradiction!

A <u>contingency</u> is neither.

Props. that (essentially) have the same truth tables are <u>logically equivalent</u> ($\Leftrightarrow$ or $\equiv$).

Ex 3, p.16

<u>Ex</u> Show $p \to q \Leftrightarrow \neg p \vee q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

must be in same order!

can combine like Rosen

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

p	q	$p \to q$	$\neg p$	q	$\neg p \vee q$
T	$\boxed{T}$	T	F	T	T
T	F	F	F	F	F
$\boxed{F}$	$\boxed{T}$	T	T	T	T
$\boxed{F}$	F	T	T	F	T

$p \to q$ F exactly when p T, q F

match, so $p \to q \Leftrightarrow \neg p \vee q$

Idea: $p \to q$ is T exactly when $\underbrace{p}_{\boxed{F}}$ <u>or</u> $\underbrace{q}_{\boxed{T}}$

"$p \Leftrightarrow q$" means that, in any particular situation, both p and q are T, or both are F (i.e., $p \leftrightarrow q$ is $\underset{\text{Think: theorem}}{\text{a tautology}}$).

<u>Ex</u> Let n be some integer.

$\underbrace{n \text{ is odd} \Leftrightarrow \text{its ones' digit is } 1, 3, 5, 7, \text{or } 9.}$

If $n = 13$, both are T.
If $n = 14$, both are F.

<u>Note 1</u> Size of truth tables

$$\begin{array}{r|ccc} & p & q & r \\ \hline \text{8 rows} \Big\{ & & & \end{array}$$

In general,

$$\begin{array}{r|cccc} & p_1 & p_2 & \cdots & p_n \\ \hline 2^n \text{ rows} \Big\{ & & & & \end{array}$$

<u>Note 2</u> Table 5 (p. 17) gives a list of basic equivalence laws, which can be used to simplify props., to show that two props. are equiv., or to show that props. are tautologies or contradictions.

as an alternative to truth tables

Don't worry now, <u>but</u> we will learn them and their twin brothers, the set identities in 1.5.

Just for kicks...

Some laws:

$\neg(\neg p) \Leftrightarrow p$

$\vee, \wedge$ are commutative and associative

$\vee$ distributes over $\wedge$ (like $\times$ over $+$)

$\wedge$ distributes over $\vee$ ($\leftarrow$ HW #5)

De Morgan's laws (HW #6 - one of them)

<u>HW Tips</u> (Relevant: pp 14-16; scan pp 17-19)

Use truth tables.

#16) Show $p \to q \Leftrightarrow \neg q \to \neg p$.

What are we proving here?

An implication is logically equivalent to its contrapositive.