1.4: SETS

The Joy of Sets

A set is a collection of objects, called elements or members.

Ex Let S denote the set $\{a, b, c\}$.

S has 3 elements: a
b
c

"$\in$" means "is a member of"
$a \in S$
$b \in S$
$c \in S$

"$\notin$" means "is not a member of"
$d \notin S$

Two sets are equal $\Leftrightarrow$ They have the same elements

Ex $\{e, o, n\} = \underbrace{\{o, n, e\} = \{n, o, n, e\}}$

the elements are e, o, and n

Writing sets

Ex (Set builder notation)

$\mathbb{Z} = \{x \mid x \text{ is an integer}\}$

= the set of all x such that ...

$= \{\underbrace{\ldots,}_{\text{ellipses: "follow the pattern"}} -2, -1, 0, 1, 2, \ldots\}$

Ex Set of all digits $= \{\underset{\text{1st}}{0}, \underbrace{1, 2}_{\text{set the pattern}}, \ldots, \underset{\text{last}}{9}\}$

If a set S contains exactly $\overset{(0,1,2,\ldots)}{n}$ distinct elements, then
1) S is a finite set
2) the cardinality of S is n $(|S| = n)$

Otherwise, S is an infinite set. Ex $\mathbb{Z}, \mathbb{R}$

Ex If $S = \{e, o, n\}$, $|S| = 3$.
Ex If $S = \{n, o, n, e\}$, $|S| = 3$. (distinct elements)

The set with no elements is the *empty set* or *null set*, denoted by "ϕ" or "$\{\}$."

$|\phi| = 0$

Ex $\{x \mid x \in \mathbb{R} \text{ and } x^2 = -1\} = \phi$

A set can have elements that are, themselves, sets.

Ex $S = \{\underbrace{\phi}_{1}, \underbrace{\{\phi\}}_{2}, \underbrace{5}_{3}, \underbrace{\{5,6\}}_{4}, \underbrace{\{\{5\},6\}}_{5}, \underbrace{\{\{6\}\}}_{6}\}$

$|S| = 6$

$\phi \in S$
$\{\phi\} \in S$
$5 \in S$
$\{5\} \notin S$

SUBSETS

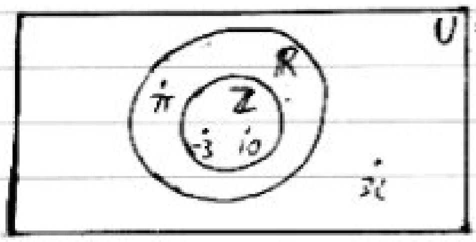
Assume A and B are sets.

A is a <u>subset</u> of B $(A \subseteq B)$ $\Leftrightarrow$
all the elements of A are also elements of B

Logic: $\forall x\,(x \in A \rightarrow x \in B)$ is T

Venn Diagram:

Ex



= universal set (maybe $\mathbb{C}$ = set of all complex #s)

$$\mathbb{Z} \subseteq \mathbb{R}$$

For any set S, 1) $\emptyset \subseteq S$
because $\forall x\,(\overbrace{x \in \emptyset}^{\text{always } F} \rightarrow x \in S)$ is T
2) $S \subseteq S$

<u>Key Math Trick</u>: If $A \subseteq B$ and $B \subseteq A$, then $A = B$
(Two sets that are subsets of each other are equal.)

A is a <u>proper subset</u> of B $(A \subset B)$ $\Leftrightarrow$
$A \subseteq B$ and $A \neq B$

<u>Ex</u> $\mathbb{Z} \subset \mathbb{R}$

POWER SETS

$P(S)$, the <u>power set</u> of a set S, is the set of all subsets of S.

<u>Ex</u> If $S = \{a, b, c\}$,

$$P(S) = \{\underbrace{\phi}_{\phi \subseteq S}, \underbrace{\{a\}, \{b\}, \{c\}}_{\text{1-elt. subsets}}, \underbrace{\{a,b\}, \{a,c\}, \{b,c\}}_{\text{2-elt. subsets}}, \underbrace{\{a,b,c\}}_{S \subseteq S}\}$$

$|S| = 3$, $|P(S)| = 8$

Rosen does later... 5.1

In general, $|P(S)| = 2^{|S|}$. Why?

① $S = \{\underset{\substack{\uparrow \\ \text{IN or} \\ \text{OUT}}}{x_1}, \underset{\substack{\uparrow \\ \text{IN or} \\ \text{OUT}}}{x_2}, \underset{\cdots}{\ldots}, \underset{\substack{\uparrow \\ \text{IN or} \\ \text{OUT}}}{x_n}\}$

You can enumerate all possible subsets by considering all possible IN/OUT combos.

$$2 \times 2 \times \ldots \times 2 = 2^n$$

2.4

② List the binary reps of integers from 0 to $2^n - 1$

	x_1	x_2	$\ldots$	x_n
ϕ:	0	0	$\ldots$	0
$\{x_n\}$:	0	0	$\ldots$	1

0: elt is not in subset
1: in

<u>Ex</u> $|P(\underbrace{\{\phi, \{\phi\}\}}_{\text{"}S\text{"}})| = 2^2 = 4$

$|S| = 2$

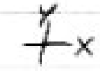
i.e., The set $\{\phi, \{\phi\}\}$ has 4 subsets, ϕ, $\{\phi\}$, $\{\{\phi\}\}$, S

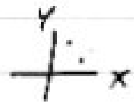
CARTESIAN PRODUCTS

Sets are unordered.

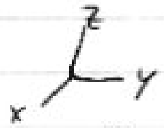
Ex $\{\{1,2\},\{2,1\}\}$

same element!

In graphing, we need ordered pairs 

Ex $\{(1,2),(2,1)\}$ 

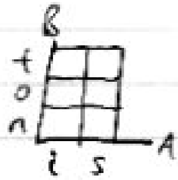
different

or ordered triples (a,b,c) 

An ordered n-tuple has the form $(x_1, x_2, \ldots, x_n)$.

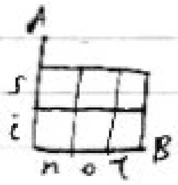
$A \times B$ = Cartesian product of sets A and B
$= \{(a,b) \mid a \in A,\ b \in B\}$

Ex $A = \{i, s\}$
$B = \{n, o, t\}$



$A \times B = \{(i,n), (i,o), (i,t), (s,n), (s,o), (s,t)\}$

$B \times A = \{(n,i), (n,s), (o,i), (o,s), (t,i), (t,s)\}$



Ex $\mathbb{R} \times \mathbb{R}$ corresponds to the standard xy-plane

Ex $\{1,3\} \times \{1,2\} = \{(1,1),(1,2),(3,1),(3,2)\}$  Can mark as you go along

$A \times B \neq B \times A$, unless $\underbrace{A = \emptyset \text{ or } B = \emptyset}_{A \times B = \emptyset}$ or $A = B$

Ex $\mathbb{Z} \times \mathbb{Z}$  lattice points

In general, $A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}$