3.3: RECURSIVE DEFINITIONS

define terms by referring to previous term(s)

Ex Give a recursive definition of the sequence $\{a_n\}$, $n \in \mathbb{Z}^+$, if $a_n = 7n - 3$.

Method 1

List some values:

$$a_1 = 7(1) - 3 = 4$$
$$a_2 = 7(2) - 3 = 11$$
$$a_3 = 7(3) - 3 = 18$$

+7, +7

Method 2 (skip)

or $\frac{a_{n+1}}{a_n}$, etc

Arithmetic vs. Geometric

$$a_{n+1} - a_n = [7(n+1) - 3] - (7n - 3)$$
$$= 7n + 7 - 3 - 7n + 3$$
$$= 7$$
$$\Rightarrow a_{n+1} = a_n + 7$$

Method 3

Think! What happens when n+1 replaces n?

$$\underset{a_n}{7n - 3} \longrightarrow \underset{a_{n+1}}{\underbrace{7(n+1)}_{\text{effect of adding 7}} - 3}$$

Recursive / Inductive defin:

$$a_1 = 4$$
$$a_{n+1} = a_n + 7 \text{ for } \underbrace{n \in \mathbb{Z}, n \geq 1}_{n \in \mathbb{Z}^+}$$

If $f(n) = 7n - 3$
$f(1) = 4$
$f(n+1) = f(n) + 7$
$\mathbb{Z}^+ \to$ for $n \in \mathbb{Z}, n \geq 1$

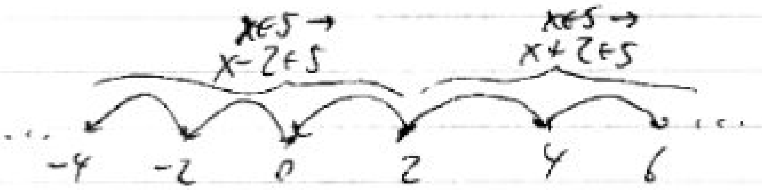
Read Exs 1-5 (203-5)

RECURSIVELY DEFINED SETS

Basic building block(s) →
if $x, y \in S$ ambiguous! or func values on BBBs

Ex ① $2 \in S$

② $\begin{cases} x - y \in S & \text{if } x \in S \text{ and } y \in S \\ x + y \in S & \text{if } \quad " \qquad " \end{cases}$ recursive/inductive step



Not in Rosen, in Transparencies →

③ Extremal clause. If something can't be included in S after a finite # of applics of ① and ②, then it is $\notin S$.

Ex The set S of bit strings with exactly one "1".

① $1 \in S$

② $\begin{cases} 0x \in S & \text{if } x \in S \\ x0 \in S & \text{if } x \in S \end{cases}$

③ Extremal clause

3.4: RECURSIVE ALGORS

Pro	Cons
Shorter	Trickier
	Base cases critical
	Memory/space considerations
	tend to repeat work