CH 6: RELATIONS

6.1

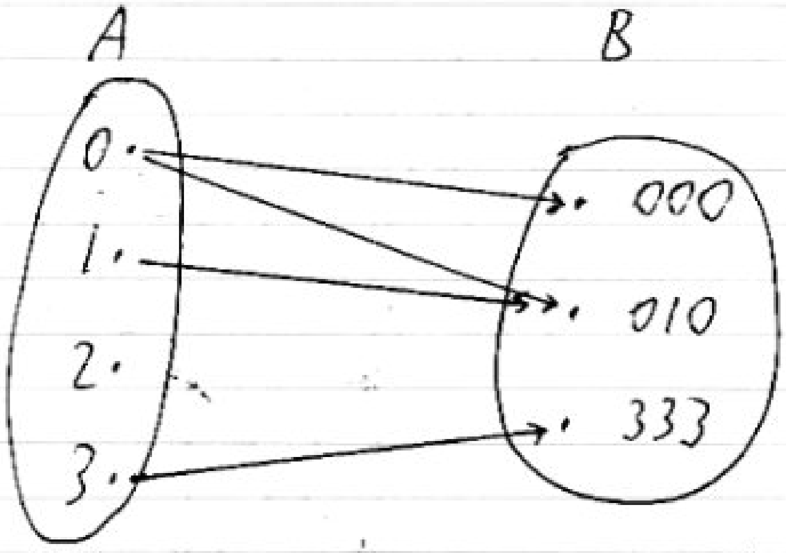
A binary relation (usually denoted by "R") from a set A to a set B is a subset of $A \times B$. It's a set of ordered pairs.

Ex $A = \{0, 1, 2, 3\}$

$B = \{000, 010, 333\}$

Let's say R is a relation from A to B s.t.
$(a, b) \in R \leftrightarrow$ digit a is in string b.

Picture of R:



Directed graph (digraph) - arrows on edges.

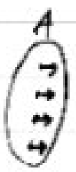
Why not?

Some relations (not this one) are functions. Our R isn't a function

0·

2· (no arrow)

Function



$R = \{ (0,000), (0,010), (1,010), (3,333) \} \subseteq A \times B$

Notation: $a R b \leftrightarrow (a,b) \in R$
$a \not{R} b \leftrightarrow (a,b) \notin R$

<u>Ex</u> $1 \, R \, 010$

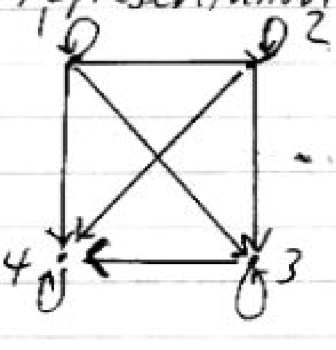
<u>Special Case</u>

R can be a relation from set A to itself.
"R is a relation on A."

<u>Ex</u> $A = \{1, 2, 3, 4\}$

$R = \{ (a_i, a_j) \mid a_i \le a_j \}$ is a relation on A

Digraph representation of R:



$R = \{ (1,1), (1,2), (2,2),$
$(1,3), (2,3), (3,3),$
$(1,4), (2,4), (3,4), (4,4) \}$

6.2: n-ary Relations

An n-ary relation on the sets: $A_1, A_2, \ldots, A_n$ is a subset of $A_1 \times A_2 \times \ldots \times A_n$ (n = "degree").

Ex (Database of students)

Fields (A_i sets)

Records - elements of R

Last Name	First Name	ID #	Major
Jones	Paul	312	CS
Smith	John	463	CS
Smith	Paul	110	Math

The ID # field is a primary key, since there are no repeats. An ID# determines a 4-tuple.

6.3: Representing Binary Relations

① Digraphs
② 0-1 Matrices

old
Ex

$$\begin{array}{cc} & \overbrace{\begin{array}{ccc} \overset{b_1}{000} & \overset{b_2}{010} & \overset{b_3}{333} \end{array}}^{B} \\ A \left\{ \begin{array}{c} 0\ a_1 \\ 1\ a_2 \\ 2\ a_3 \\ 3\ a_4 \end{array} \right. & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

m_{ij} = entry in row i, column j $= \begin{cases} 1 \leftrightarrow a_i R b_j \\ 0 \leftrightarrow a_i R b_j \end{cases}$

Old Ex

$A = \{1, 2, 3, 4\}$

$R = \{(a_i, a_j) \mid a_i \leq a_j\}$ is a relation on A

① Digraph ✓

② 0-1 Matrix Rep.

must be in same order

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \multicolumn{4}{c}{\left[\begin{matrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{matrix}\right]} \\ 2 & \\ 3 & \\ 4 & \end{array}$$

6.1/6.5: Equivalence Relations

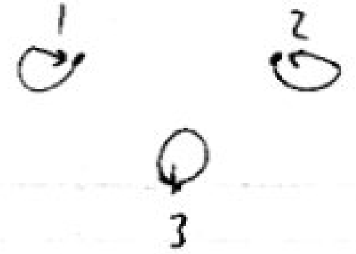
Let's say R is a relation on a set A.

R is reflexive $\longleftrightarrow$ $\forall a \in A \ (a,a) \in R$
"aRa"

Ex $A = \{1,2,3\}$

R is reflexive $\leftrightarrow$ the corresponding digraph has at least

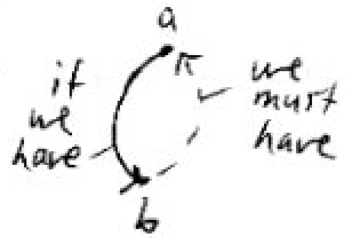
more edges are OK



Property is implied in English usage.

R is symmetric $\leftrightarrow [\forall a,b \in A \ (aRb \rightarrow bRa)]$

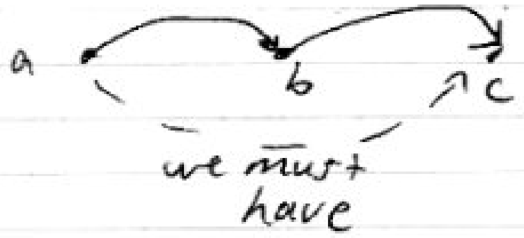
Ex



R is <u>transitive</u> $\leftrightarrow$

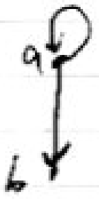
$$[\forall a,b,c \in A\ (aRb \text{ and } bRc \rightarrow aRc)]$$

<u>Ex</u> If we have



R is an <u>equivalence relation</u> $\leftrightarrow$
it is reflexive, symmetric, and transitive

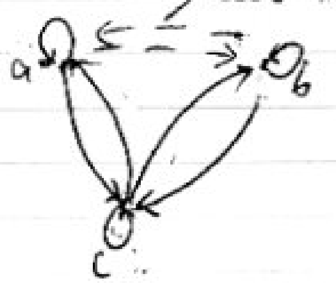
<u>Ex</u>



<u>NO</u>
not reflexive (no $\circlearrowleft_b$)

not symmetric (we have $\overset{a}{\downarrow}_{b}$, so we need $\overset{a}{\uparrow}_{b}$)

<u>Ex</u> need for transitivity
<u>NO</u>



In an EC, every elt is related to every other.

<u>Ex</u>

<u>YES</u>

(E.R.)
In an equivalence relation, the elements of an equivalence class are all related to one other and themselves.

Ex $A = \mathbb{Z}$
$R = \{(a,b) \mid a \equiv b \pmod 3\}$ is an E.R. on A.

$$\mathbb{Z}: \left| \begin{array}{c|c|c} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 0 & 1 & 2 \\ -3 & -2 & -1 \\ -6 & -5 & -4 \end{array} \right|$$

3 ECs

Ex A = freshmen at a college (forced into apts.)
$R = \{(a,b) \mid a$ lives in the same apt. as $b\}$ is an E.R. on A.

Why is it R? S? T?

ECs correspond to the apts.

evil twin