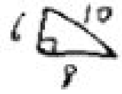
Critical # Th'y Theorem:
Fermat's Last Theorem (Conjecture ~350 yrs. old)
Proven by Andrew Wiles at Princeton.

$$x^2+y^2=z^2$$

Find $x,y,z \in \mathbb{Z}^+$ that solve this.
There are many triples:

$$3^2+4^2=5^2$$ 



$$6^2+8^2=10^2$$

$$5^2+12^2=13^2$$

$$8^2+15^2=17^2$$

$$\vdots$$

∞ many triples

Thm: for $n \in \mathbb{Z}, n \geq 3$

1983- Gerd Faltings proved: finitely many sol'n.

$$x^n+y^n=z^n$$

has no solution (x,y,z)
where $x,y,z \in \mathbb{Z}^+$

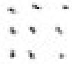
$$x^3+y^3=z^3$$
$$x^4+y^4=z^4$$
$$\vdots$$

Fermat wrote in a margin

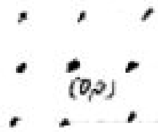
A Personal Favorite

Consider the $\mathbb{Z}\times\mathbb{Z}$ lattice of pts.

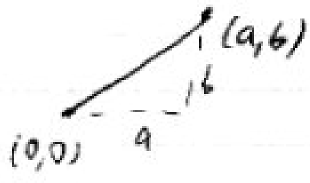
$\quad \| $ set of all pts. (x,y) both integer



Is there a straight line that passes through $(0,0)$ and no other lattice point?



Let's assume we have a straight line that hits $(0,0)$ and some other lattice point (a,b).



The slope must be $\frac{b}{a} \ni \in \mathbb{Z}$ or undefined $\Updownarrow$

The slope must be rational ($\in \mathbb{Q}$). or undefined.

We've shown
If a straight line hits $(0,0)$ and another lattice point (a,b), then its slope is rational or undefined.

The contrapositive must be true:

If the slope of a line is irrational, then the straight line can't hit both $(0,0)$ and another lattice pt.

<u>Ex</u> $y = \pi x$

<u>Ex</u> $y = \sqrt{2}\, x$

$\vdots$

Infinitely many lines, each corresp. to an irrat'l #.