# 15.3: MOTION

When $\vec{r}$ is twice diff'e,

$\vec{r}(t)$ = the position vector of a point $P$ at $t$

Imagine $P$ moving along $C$ as $t \nearrow$.

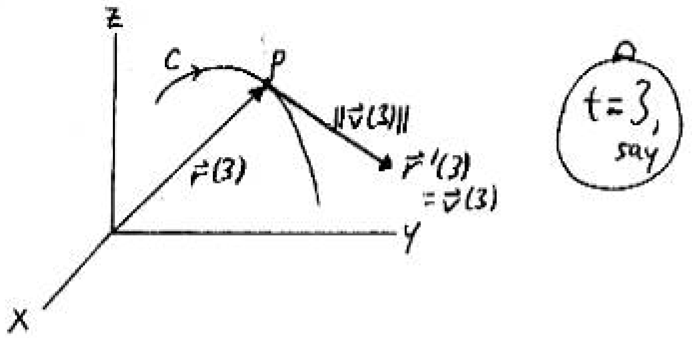
$\vec{r}\,'(t) = \vec{v}(t)$
= the velocity [vector] of $P$ at $t$

A vector is characterized by...

It's a tangent vector that indicates direction of motion.

You can't just look at C and determine the length of $\vec{v}(t)$.

Its length depends on how $C$ is parameterized. What is the significance of its length?




$\|\vec{r}\,'(t)\| = \|\vec{v}(t)\|$
= the speed of $P$ at $t$

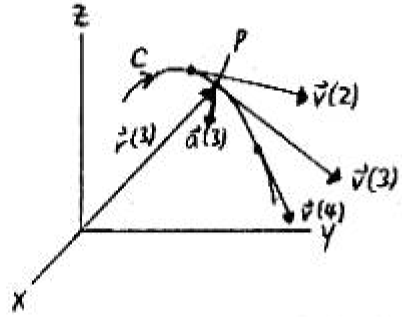
Why? $\|\vec{v}(t)\| = \left\| \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle \right\|$

$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$

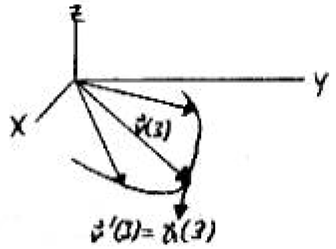
$= \frac{ds}{dt}$, the rate of change of arc length with respect to time $\Big\}$ SPEED!!

$\vec{r}\,''(t) = \vec{v}\,'(t) = \vec{a}(t)$
$= $ the <u>acceleration [vector]</u> of $P$ at $t$

Graph of $\vec{r}$




Graph of $\vec{v}$




From $P$, $\vec{a}(t)$ typically points into the concave side of $C$.

twisted cubic $\langle at, bt^2, ct^3 \rangle$ $a, b, c$ all non-0

<u>Ex</u> Let $\vec{r}(t) = \langle t, t^2, t^3 \rangle \Rightarrow \vec{r}(3) = \langle 3, 9, 27 \rangle$
$\vec{v}(t) = \langle 1, 2t, 3t^2 \rangle \Rightarrow \vec{v}(3) = \langle 1, 6, 27 \rangle$ *
$\vec{a}(t) = \langle 0, 2, 6t \rangle \Rightarrow \vec{a}(3) = \langle 0, 2, 18 \rangle$

$$* \ \|\vec{v}(3)\| = \sqrt{(1)^2 + (6)^2 + (27)^2} = \sqrt{766} \approx 27.7$$

<u>Note</u> $C$ lies on the intersection of the cylinders $y = x^2$, $z = x^3$. (p. 750)

<u>Ex</u> $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$, $t$ in $\mathbb{R}$.
$C$ is the helix from 15.1.3.

$$\begin{aligned}\text{Speed} &= \|\vec{v}(t)\| \\ &= \|\langle -\sin t, \cos t, 1\rangle\| \\ &= \sqrt{\underbrace{(-\sin t)^2 + (\cos t)^2}_{=1} + (1)^2} \\ &= \sqrt{2} \quad (\text{constant})\end{aligned}$$

$\vec{v}(t)$ always has length $\sqrt{2}$

The particle travels $\sqrt{2}$ units for every unit increase in $t$.

<u>Ex</u> $\vec{r}(t) = \langle \cos(t^3), \sin(t^3), t^3 \rangle$, $t$ in $\mathbb{R}$.
Same $C$, but...

$$\begin{aligned}\text{Speed} &= \|\vec{v}(t)\| \\ &= \|\langle -3t^2\sin(t^3), 3t^2\cos(t^3), 3t^2\rangle\| \\ &= \|3t^2\langle -\sin(t^3), \cos(t^3), 1\rangle\| \\ &= |\underbrace{3t^2}_{\substack{\geq 0 \\ \text{for all } t}}| \sqrt{\underbrace{[-\sin(t^3)]^2 + [\cos(t^3)]^2}_{=1} + 1} \\ &= 3t^2\sqrt{2} \\ &= 3\sqrt{2}\,t^2\end{aligned}$$

We lose smoothness at $t=0$, since $\vec{r}(0) = \vec{0}$

| | | $\vec{v}(t)$ |
|---|---|---|
| ↑ Speeds up | $(t \to \infty)$ | lengthens |
| ↑ Slows down | $(t \to 0^-)$ | shrinks |

<u>Ex</u> Given $\vec{r}(0)$ ("starting pt."), $\vec{v}(t)$ (describes motion) $\Rightarrow$ Find $\vec{r}(t)$ (See 15.2.2)