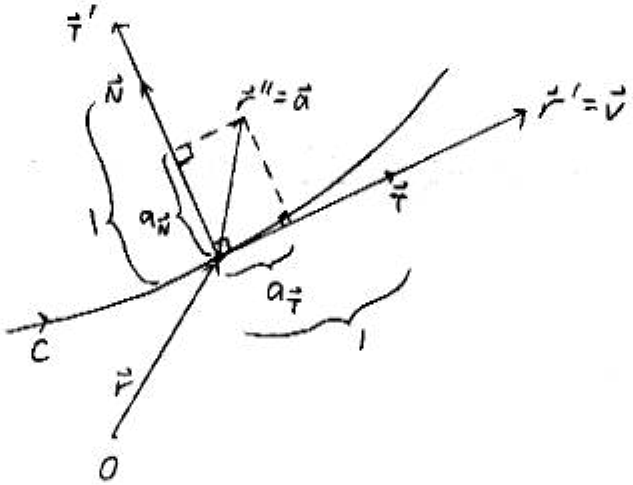
15.5: UNDERSTANDING $\vec{a}$ (Bonus)

$$
\begin{tikzcd}
\vec{r} \arrow[d, "D_t"'] & & \\
\vec{r}\,' = \vec{v} \arrow[rr, "\text{Norm.}"] \arrow[d, "D_t"'] & & \vec{T} \arrow[d] \\
\vec{r}\,'' = \vec{a} & & \vec{T}\,' \arrow[r, "\text{Norm.}"] & \vec{N}
\end{tikzcd}
$$

tangent
$\|\vec{v}\| = \text{speed}$

points into concave side

$\vec{T}\,', \vec{N}$ also do indicate direction of turn



Larson 6ed p. 797: $\vec{a}$ lies in plane of $\vec{T}, \vec{N}$

$\vec{a}(t) = a_{\vec{T}}\vec{T}(t) + a_{\vec{N}}\vec{N}(t)$

$\|\vec{v}\| = \text{speed} = \frac{ds}{dt} =$ rate of change of arc length wrt t

What gives us rate of change of speed, $\frac{d^2s}{dt^2}$?
(What plays the role that "acceleration" played in Calc I?)

$\|\vec{a}\| = K = \|\vec{r}\,''(s)\|$ if ALP

<u>Not</u> $\|\vec{a}\|$

<u>Is</u> $\text{comp}_{\vec{T}}\,\vec{a} = a_{\vec{T}} =$ <u>tangential component of acc.</u>

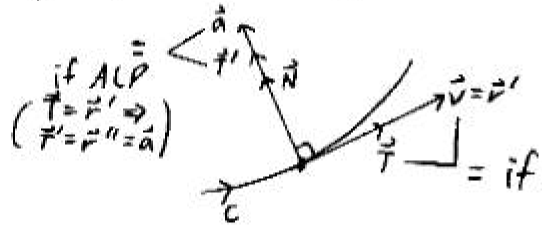
$$= \frac{\vec{a} \cdot \vec{T}}{\|\vec{T}\|} \leftarrow = 1$$

$$= \vec{a} \cdot \vec{T}$$

depends on speed, not K

See 15.4.2 Note

<u>If</u> speed is constant, $\|\vec{v}\| = c \Rightarrow \vec{v} \perp \vec{a}$ by Sphere Thm.



$\vec{a} \perp \vec{T}$
$a_{\vec{T}} = \vec{a} \cdot \vec{T}$

$a_{\vec{T}} = 0$ (reflects constant speed)

<u>What is $a_{\vec{N}}$?</u>

$a_{\vec{N}} =$ <u>normal (centripetal) component of acc.</u>

$= \vec{a} \cdot \vec{N}$

$\overset{\text{turns out}}{=} K \left(\frac{ds}{dt}\right)^2$ ← speed

depends on speed and K

$a_{\vec{N}}$ is the "barf factor," even if no ALP.
High if crazy driver (goes fast when C curvy).

If speed, K, driver are high.

<u>Note 1</u>: $a_{\vec{N}} \geq 0$, because ① $\vec{a}, \vec{N}$ both point into concave side
② $K \geq 0$, $\left(\frac{ds}{dt}\right)^2 \geq 0$

K is barf factor if ALP

<u>Note 2</u>: <u>If</u> ALP, $\|\vec{a}\| = a_{\vec{N}} = K$ (We got <u>some</u> meaning for $\|\vec{a}\|$.)

$\vec{a}, \vec{N}$ point in same direction (see figure)

speed = 1