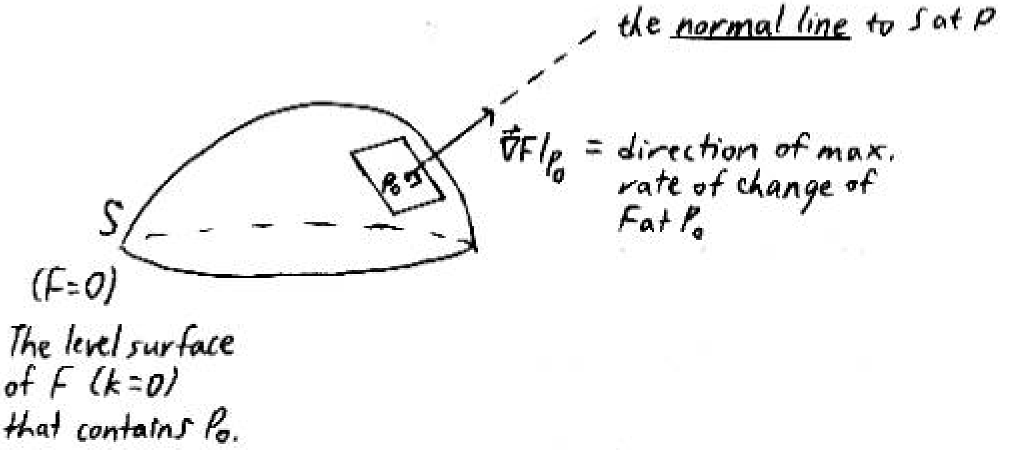
16.7: TANGENT PLANES and NORMAL LINES

Let S be the graph of $F(x, y, z) = 0$.
Let $P_0\,(x_0, y_0, z_0)$ be a point on S.

If $\vec{\nabla} F = \langle F_x, F_y, F_z \rangle$ is cont.,

then $\underset{\substack{\uparrow \\ \text{at}}}{\vec{\nabla} F|_{P_0}} \perp$ (the tangent plane to S at P_0).

#27: Every normal line to a sphere passes through the center.



What's an eq. for the tangent plane at P_0?

Ingredients

A point: $P_0\,(x_0, y_0, z_0)$
A normal: $\vec{\nabla} F|_{P_0} = \langle F_x|_{P_0}, F_y|_{P_0}, F_z|_{P_0} \rangle \leftarrow$ "$\vec{n}$"

From 14.5,

$$\boxed{(F_x|_{P_0})(x - x_0) + (F_y|_{P_0})(y - y_0) + (F_z|_{P_0})(z - z_0) = 0}$$

Ex ⓐ Find an eq. for the tangent plane to the graph of $z = 2x^2+y^2$ at $P_0\,(2,3,17)$. ← from 16.6
ⓑ Find eqs. for the normal line at P_0.

Sol'n

$$z = 2x^2+y^2$$

Isolate 0 on one side.

$$0 = \underbrace{2x^2+y^2-z}_{=F(x,y,z)}$$

$$\vec{\nabla}F = \langle 4x,\ 2y,\ -1\rangle$$

$$\vec{\nabla}F\Big|_{P_0} = \langle 4(2),\ 2(3),\ -1\rangle$$

$$= \langle 8, 6, -1\rangle \quad \leftarrow \text{"}\vec{n}\text{"}$$

Projection in xy-plane is $8\vec{i}+6\vec{j} = \vec{\nabla}f(2,3)$, where $f(x) = 2x^2+y^2$.

ⓐ Tangent plane

$$\boxed{8(x-2) + 6(y-3) - (z-17) = 0}$$

ⓑ Normal line

$$\boxed{\begin{cases} x = 2 + 8t \\ y = 3 + 6t \\ z = 17 - t \end{cases}, \quad t \text{ in } \mathbb{R}}$$

$\uparrow$ P_0 $\quad$ $\uparrow$ $\vec{n}$