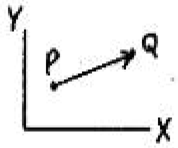
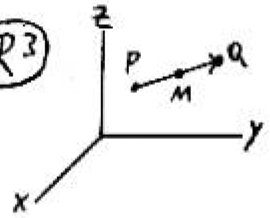
REVIEW: CH.14

VECTORS

$\mathbb{R}^2$



$\mathbb{R}^3$



$\vec{PQ} = \langle \Delta x, \Delta y \rangle$

$\vec{i}, \vec{j}$

$\vec{PQ} = \langle \Delta x, \Delta y, \Delta z \rangle$

$\vec{i}, \vec{j}, \vec{k}$

Midpoint M: avg. coords.

$\|\vec{v}\| = \sqrt{\sum v_i^2}$

$\vec{v} + \vec{w}$, $c\vec{v}$

Compute, Draw

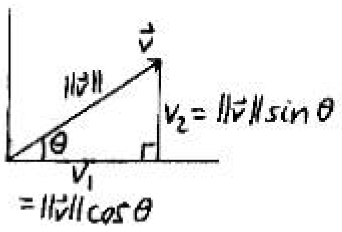
$\vec{w} \parallel \vec{v} \Leftrightarrow \exists c: \vec{w} = c\vec{v}$, or $\vec{v} = \vec{0}$

there exists

$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$

Horiz., Vert. Components

$\mathbb{R}^2$



$\tan\theta = \frac{v_2}{v_1}$ $(\ne 0)$

$0 \le \theta \le \pi$

Vector Properties, Proofs

GRAPHS in $\mathbb{R}^3$

Spheres

CTS $\Rightarrow (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$ 

Graph, Describe Region R in words $\Leftrightarrow$ Describe R using symbols

$\vec{a}\cdot\vec{b}$ (scalar) — Properties, Proofs — **$\vec{a}\times\vec{b}$ (vector)**

$\vec{a}\cdot\vec{b} = \sum a_i b_i$

$\vec{a}\times\vec{b} = \begin{array}{c|ccc|l} & & \ominus & & \\ & \vec{i} & \vec{j} & \vec{k} & \\ & & & & \leftarrow \vec{a} \\ & & & & \leftarrow \vec{b} \end{array}$

$\vec{a}\cdot\vec{b} = \|\vec{a}\|\,\|\vec{b}\|\cos\theta$ ($\star$)

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|}\right) \leftarrow$ if $\neq 0$

$\|\vec{a}\times\vec{b}\| = \|\vec{a}\|\,\|\vec{b}\|\sin\theta$

$=$ Area of 

Direction of $\vec{a}\times\vec{b}$

$\perp \vec{a}, \vec{b}$

Right-Hand Rule

$\vec{a}\perp\vec{b} \Leftrightarrow \vec{a}\cdot\vec{b} = 0$

$\vec{a}\parallel\vec{b} \Leftrightarrow \vec{a}\times\vec{b} = \vec{0}$

Physics

Work $W = \vec{F}\cdot\vec{d}$

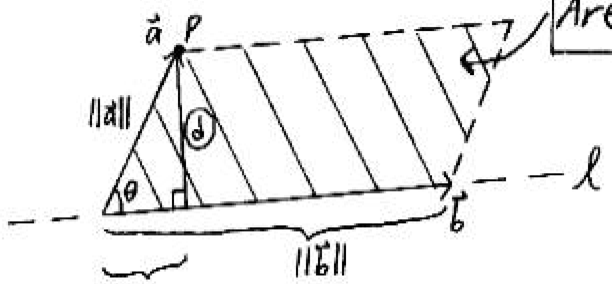
Torque uses "×" (Optional)

Inequalities

Cauchy-Schwarz $|\vec{a}\cdot\vec{b}| \le \|\vec{a}\|\,\|\vec{b}\|$ (from ⊛)

Triangle $\|\vec{a}+\vec{b}\| \le \|\vec{a}\| + \|\vec{b}\|$

Geometry



Area $= \|\vec{a}\times\vec{b}\|$ ($\frac{1}{2} \Rightarrow$ triangle)

ⓓ = distance bet. P, ℓ

$$= \frac{\|\vec{a}\times\vec{b}\|}{\|\vec{b}\|}$$

$\text{comp}_{\vec{b}}\vec{a} = \|\vec{a}\|\cos\theta$
(scalar)

$$= \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|} \quad \text{from ⊛} \quad (-\ \ <0)$$

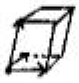
$\text{proj}_{\vec{b}}\vec{a} = (\text{comp}_{\vec{b}}\vec{a})\frac{\vec{b}}{\|\vec{b}\|}$
(vector)

$$= \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|^2}\,\vec{b}$$

TSP $= (\vec{a}\times\vec{b})\cdot\vec{c}$

$= \vec{a}\cdot(\vec{b}\times\vec{c})$

$$= \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} \begin{array}{l} \leftarrow \vec{a} \\ \leftarrow \vec{b} \\ \leftarrow \vec{c} \end{array}$$

$|TSP|$ = Volume of box  (= 0 ⇔ coplanar)

TVP (I'll give)

LINES

Vector Eq. for ℓ:

$$\overrightarrow{OP} = \overrightarrow{OP_1} + t\vec{a}, \ t \text{ in } \mathbb{R}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} t \quad '' \ ''$$

$\Rightarrow$ Parametric Eqs.

$$\begin{cases} x = x_1 + a_1 t \\ y = y_1 + a_2 t \\ z = z_1 + a_3 t \end{cases}, \ t \text{ in } \mathbb{R}$$

$\Rightarrow$ Symmetric Eqs.

$$\underbrace{\frac{x - x_1}{a_1}}_{\substack{\text{If } a_1 = 0 \\ \Rightarrow x = x_1}} = \frac{y - y_1}{a_2} = \frac{z - z_1}{a_3}$$

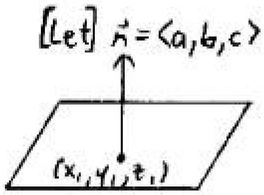
$\times \leftarrow$ Solve system for (t, u)

$\Downarrow$ Use either

(x, y, z)

Use direction vectors for angles, $\parallel$, $\perp$.

PLANES

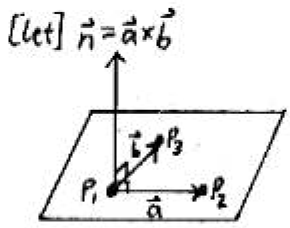


$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$$

$$ax + by + cz + d = 0 \Longrightarrow \vec{n} = \langle a, b, c \rangle$$

(or any non-$\vec{0}$ scalar mult.)

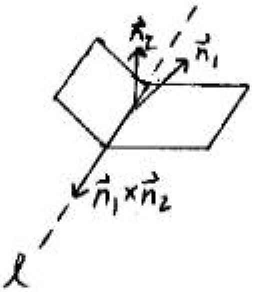
Use normal vectors for angles, $\parallel$, $\perp$.



Use $\vec{n}$ and P_1, say, to construct an eq. for the plane.

Find ℓ

Plane 2 looks like $z=k$; plug in $x=0$ here?



Find a point.

Plug $x = 0$, say, into the system.

Find a direction vector.

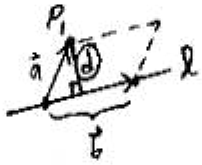
$\vec{n}_1 \times \vec{n}_2$ works.

DISTANCES

2 Points

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

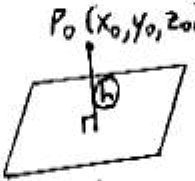
Point, Line



$$d = \frac{\|\vec{a} \times \vec{b}\|}{\|\vec{b}\|}$$ $\leftarrow \vec{b} \parallel \ell$ (Careful! I sometimes do $\vec{b}$ $\vec{a}$ to ensure $\vec{n}$ points "up.")

Point, Plane

<u>If know eq. of plane</u>

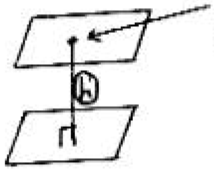


$$\underbrace{ax + by + cz + d = 0}_{A}$$

$$h = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$ $\leftarrow$ Plug P_0 into A

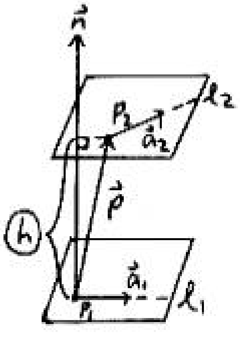
<u>If not, find one!</u>

2 $\parallel$ Planes



Find a point.
Plug in $x = 0, y = 0$, say.

Skew Lines



Let $\vec{n} = \vec{a}_1 \times \vec{a}_2$.

$$h = |\text{comp}_{\vec{n}}\, \vec{p}| = \frac{|\vec{p} \cdot \vec{n}|}{\|\vec{n}\|}$$

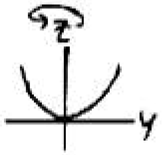
SURFACES

Traces

Spheres
Planes
Cylinders
Ex 1 variable missing $\Rightarrow$ sweep ($\parallel$ axis)

Surfaces of Revolution

Ex



$z = y^2$ in yz-plane

Replace y^2 with $y^2 + x^2$.
(x: missing variable)
Don't touch "axis variable" (z, here).

Quadric Surfaces

Know Basic Eqs., Axes for 6 Basic Types
Surfaces of Revolution may help!
Traces

Tricks: Variable-Switch
$\Rightarrow$ may change axis
Coefficients
$\Rightarrow$ makes it easier to identify