REVIEW: CH. 15, 16.1, 16.2

15.1: VVFs

Smooth curve determined by VVF $\vec{r} : D \to V_n$
$D \subseteq \mathbb{R}$

$\vec{r}\,'$ cont., never $\vec{0}$ on I
(except maybe at endpoints)

$$\vec{r}(t) = \langle f(t), g(t), \underbrace{h(t)}_{\text{if in } V_3} \rangle$$

$$\text{Arc Length "}L\text{"} = \int_a^b \underbrace{\|\vec{r}\,'(t)\|}\,dt$$

$$= \underset{\text{"speed"}}{\|\vec{v}(t)\|} = \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}$$

$$= \frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

Use Table of Integrals?

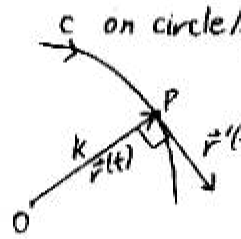
15.2: LIMITS, D_t, $\int dt$, CONTINUITY for VVFs

Do component-by-component.
Apply FTC
Solve Differential Eqs.

D_t Rules

Linearity
Product Rules for $\cdot$, $\times$, $f\vec{u}$
Chain Rule for $\vec{u}(f(t))$

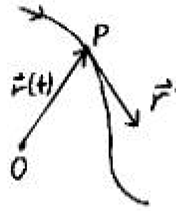
Sphere Theorem



If $\|\vec{r}(t)\| = k$ on I, r diff'e

$\Rightarrow \vec{r}\,'(t) \perp \vec{r}(t)$ on I

15.3: MOTION



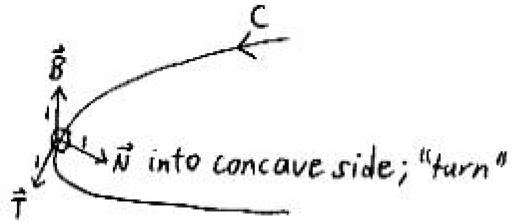
$\vec{r}\,'(t) = \vec{v}(t)$, a tangent vector indicating direction of motion of P

$\|\vec{v}(t)\|$ = speed of P

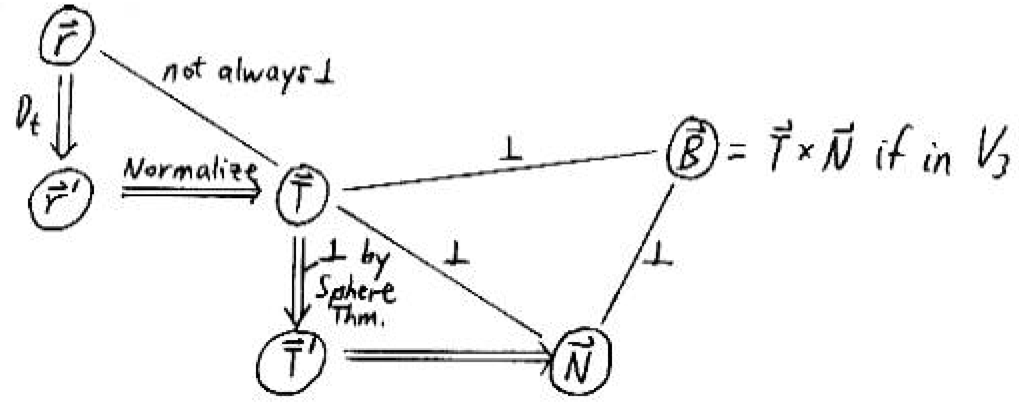
$\vec{r}\,''(t) = \vec{v}\,'(t) = \vec{a}(t)$

15.4/15.5

TNB Frame

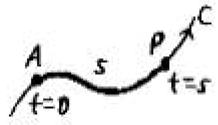


Point P, $\vec{T}$, $\vec{N}$ $\Rightarrow$
Get osculating plane at P.



ALPs

$\vec{r}(s)$



$$\begin{aligned} \text{Speed} &= \|\vec{v}(s)\| \\ &= \|\vec{r}'(s)\| \\ &= 1 \end{aligned}$$

$$\Rightarrow \vec{T}(s) = \vec{r}'(s)$$

κ

$\rho = \frac{1}{\kappa}$  for $\mathbb{R}^2$

If ALP,

$$\kappa(s) = \|\vec{r}\,''(s)\| = \|\vec{T}\,'(s)\| = \left\| \frac{d\vec{T}}{ds} \right\|$$

$\vec{T} = \vec{r}\,'$ How quickly is $\vec{T}$ changing direction?

If not ALP,

$$\kappa(t) = \frac{\|\vec{T}\,'(t)\|}{\|\vec{r}\,'(t)\|} \quad \left(= \left\| \frac{d\vec{T}/dt}{ds/dt} \right\| \right)$$

$$= \frac{\|\vec{v}(t) \times \vec{a}(t)\|}{\|\vec{v}(t)\|^3} \quad \text{or} \quad \frac{\|\vec{r}\,'(t) \times \vec{r}\,''(t)\|}{\|\vec{r}\,'(t)\|^3}$$

If a plane curve: $y = g(x)$,

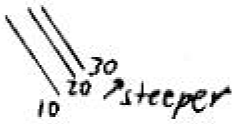
$$\kappa(x) = \frac{|y''|}{[1 + (y')^2]^{3/2}}$$ (I'll give.)

16.1: $z = f(x,y)$, etc.

$f : D \to \mathbb{R}$, where Domain $D \subseteq \mathbb{R}^n$
Find D.

Level Curves

Graph $k = f(x,y)$ in for several k-values.



Level Surfaces

Graph $k = f(x,y,z)$ in
Identify them.

16.2: LIMITS and CONTINUITY

"$\lim_{(x,y)\to(a,b)} f(x,y) = L$" means that

$\forall \epsilon > 0, \exists \delta > 0$ such that

<u>if</u> $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$

<u>then</u> $|f(x,y) - L| < \epsilon$

Computing Limits

Basic Laws, Direct Sub
Dealing with $\frac{0}{0}$
Polar Coordinates

When Limits DNE

Two-Path Rule to show $\nexists$ Ex $\downarrow\swarrow$ different limits

(a,b)

f is cont. at (a,b) if $\displaystyle\lim_{\substack{(x,y)\to(a,b)\\ \text{through } D}} f(x,y) = f(a,b)$ [and these exist].