REVIEW: 16.3 - 16.9

16.3: PDs

1st - Order Ex

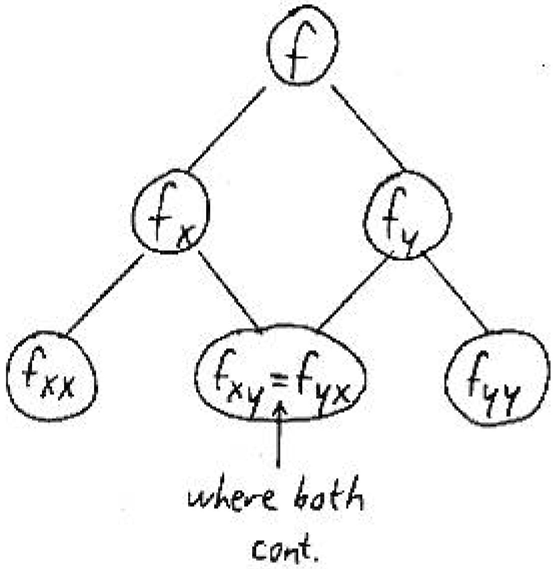
$$f_x(x,y) \text{ or } \frac{\partial f}{\partial x}(x,y)$$

$$= \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

To find: Treat y as constant,
Differentiate wrt x.

2nd - Order Exs

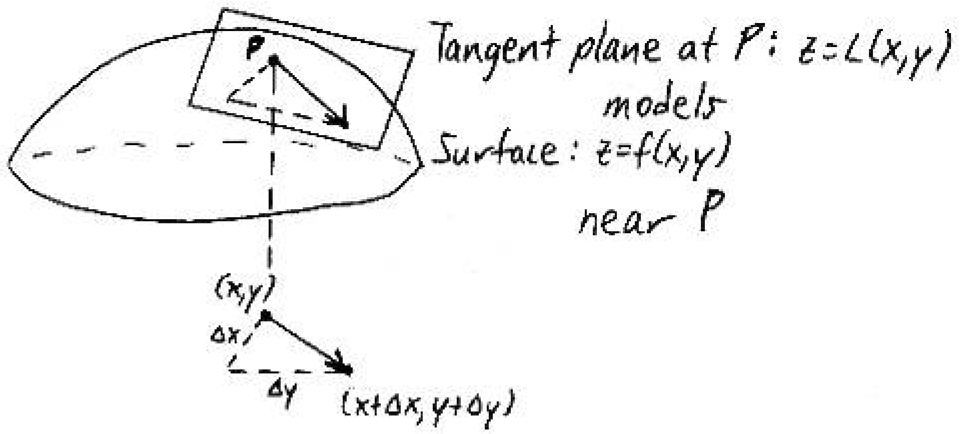
$$f_{xx} = (f_x)_x = \frac{\partial^2 f}{\partial x^2}$$
$$\overrightarrow{f_{xy}} = (f_x)_y = \overleftarrow{\frac{\partial^2 f}{\partial y \partial x}}$$



16.4: INCREMENTS and DIFFERENTIALS

$\Delta x, \Delta y, \Delta z$, etc. $\qquad\qquad$ dx, dy, dz, etc.

Used to find linear approxs. for f near a seed point $P(x,y,f(x,y))$.



$dx = \Delta x = \text{new } x - \text{old } x$
$dy = \Delta y = \text{new } y - \text{old } y$

dz = change in z along tangent plane

$= (x \text{ slope})(x \text{ run}) + (y \text{ slope})(y \text{ run})$ ← Idea: rise = (slope)(run)

$= \quad f_x(x,y)\ dx \quad + \quad f_y(x,y)\ dy$

approximates Δz, the actual change in z along the surface

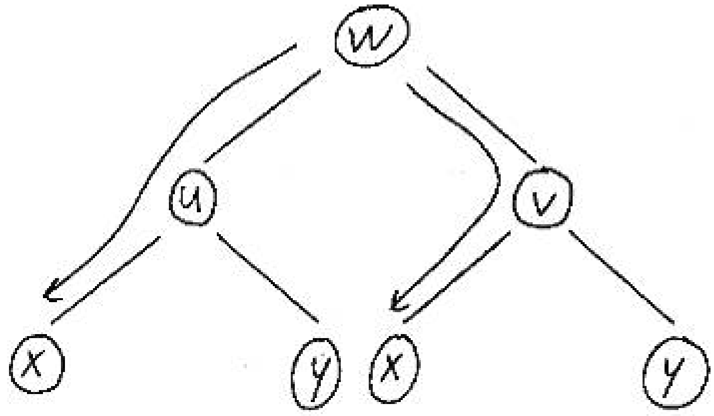
We approx. $f(x+\Delta x, y+\Delta y) = f(x,y) + \Delta z$
by $L(x+\Delta x, y+\Delta y) = f(x,y) + dz$.

<u>Theory Notes</u>

f_x, f_y cont. on open region $\Rightarrow f(x,y)$ diff'e there
f diff'e at $(a,b) \Rightarrow f$ cont. there

16.5: CHAIN RULES

Ex

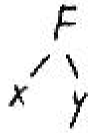


$$\frac{\partial w}{\partial x} = \underbrace{\underbrace{\overbrace{\frac{\partial w}{\partial u}}^{\text{At end, write in terms of } x,y}\frac{\partial u}{\partial x}}_{\text{Product along path } w\to x}}_{} + \underbrace{\overbrace{\frac{\partial w}{\partial v}}\frac{\partial v}{\partial x}}_{}$$

Add these path products.

If $F(x,y)=0$ describes a diff'e func. f such that $y=f(x)$,

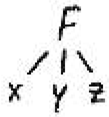
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad \left(\text{“Negative reciprocal”}\right)$$



← treat as indep.

If $F(x,y,z)=0$ " " $z=f(x,y)$,

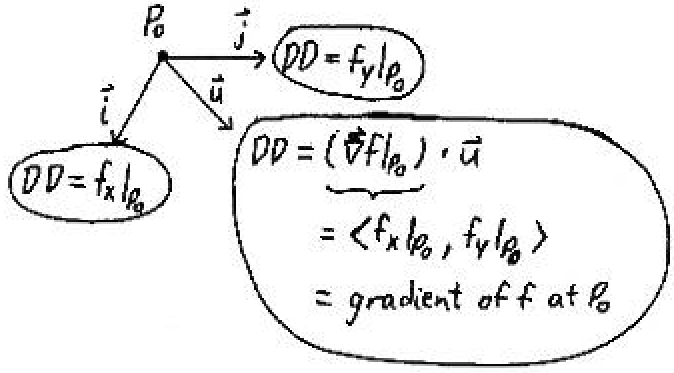
$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

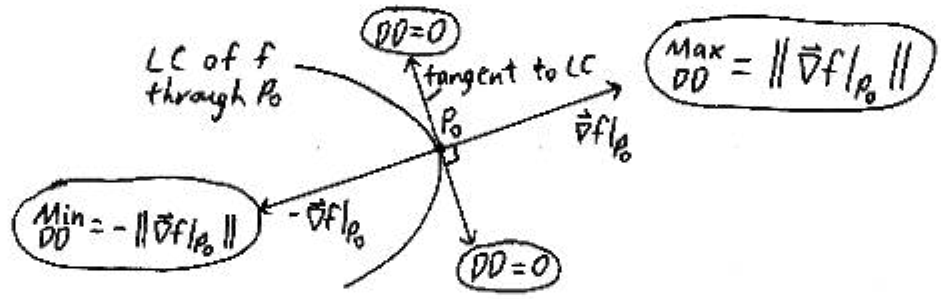


← treat as indep.

16.6 : DDs / 16.7

$f(x,y)$



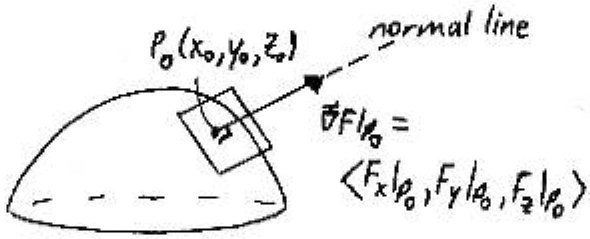


$f(x,y,z)$

Level curves (LCs) → Level surfaces (LSs)
Tangent line → $\underbrace{\text{Tangent plane}}$

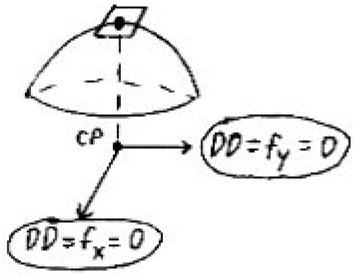
Eq.: $(F_x|_{P_0})(x-x_0) + (F_y|_{P_0})(y-y_0) + (F_z|_{P_0})(z-z_0) = 0$

Ideas analogous to $f(x,y)$
<u>Ex</u> $\vec{\nabla} f \perp$ (Tangent to LC/LS)



16.8: OPTIMIZATION I

CPs are the only places where L. Max./Min. can occur.



(a,b) is a CP $\Leftrightarrow$

⓪ (a,b) in Dom(f)

① $\vec{\nabla} f(a,b) = \langle f_x(a,b), f_y(a,b) \rangle = \vec{0}$

or ② DNE

2nd Derivative Test to Classify CPs

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} \Big\} \text{Assume all cont.}$$

$$= f_{xx} f_{yy} - (f_{xy})^2$$

At a CP where $\vec{\nabla} f = \vec{0}$,

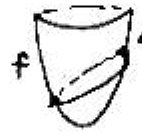
① If $D > 0$,

(1a) If f_{xx} (or f_{yy}) < 0 ☹ $\Rightarrow$ L. Max.
(1b) $>$ ☺ $\Rightarrow$ L. Min.

② If $D < 0 \Rightarrow$ Saddle Pt.

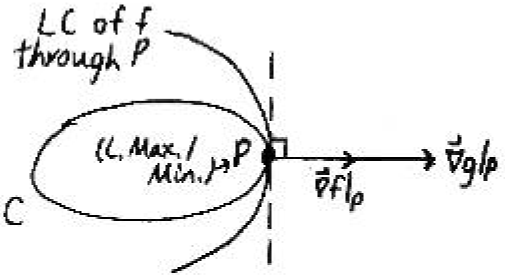
③ If $D = 0 \Rightarrow$ No info

16.9: CONSTRAINED OPTIMIZATION – LAGRANGE MULTIPLIERS (LMs)

Ex 

← Find L. or A. Max./Min. [Pts.] of f along [the image of] C.


$C : g(x,y) = 0$



$\Rightarrow \vec{\nabla} f \parallel \vec{\nabla} g$ at P

$\Rightarrow \vec{\nabla} f = \lambda \vec{\nabla} g$ at P for some $\underset{\uparrow \atop LM}{\lambda}$

Solve $\begin{cases} \vec{\nabla} f(x,y) = \lambda \vec{\nabla} g(x,y) \\ g(x,y) = 0 \end{cases}$ for (x, y, λ)

↑ can differ among (x,y) candidates; don't have to find

Evaluate f at the candidates, and compare.

↑ See Strategies for Classifying, Solving Systems.