# REVIEW: CH.17

## 17.1/17.2: ∬s

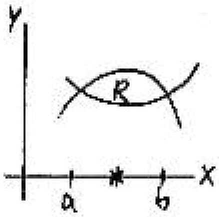
$\iint_R 1\, dA = \text{Area of } R$

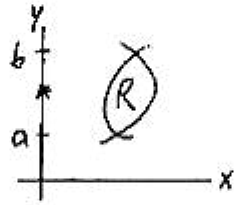
$dA = dx\,dy$ or $dy\,dx$

### ① Sketch R

?

Solve systems to locate intersection points.





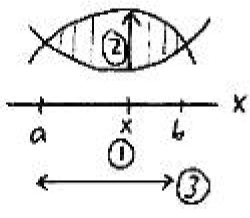
Identify which graph is on top vs. bottom
or right vs. left.

Use test values? *

If  ⇒ Split R.

### ② Set up limits of ∬

Ex



① Aim outer var

can't depend on

$$\int_{x=a}^{x=b} \int_{y=bot}^{y=top} 1 \; dy \; dx$$

② Shoot inner var

③ Slide/Scan along outer var

## Reversing Order of $\iint$

Why? If R-scan is awkward, or
if hard to do inner $\int$.

If $\int_{x=a}^{x=b} \int_{y=c}^{y=d} \sim dy\, dx$

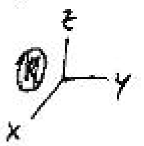
Can if all limits are constant

Otherwise, Sketch R
Switch scan (slide)/outer/aim variable
in analysis.

$\underbrace{\iint_R f(x,y)\, dA}_{\text{if} \geq 0 \text{ on } R}$ = Volume between f graph, xy-plane over R.


$\iint_R [\underset{\text{top}}{f(x,y)} - \underset{\text{bottom}}{g(x,y)}]\, dA$ = Volume between f, g graphs "over" R.
(below OK)


Can shift perspective




$dA = dx\,dz$

## 17.3: PCs (Polar)

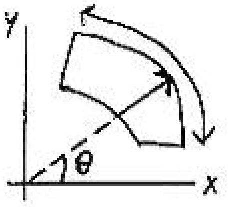
$$\iint_R f(x, y)\, dA$$

$R$: Sketch! $x$: $r\cos\theta$ $y$: $r\sin\theta$ $dA$: $r\,dr\,d\theta$ (or $r\,d\theta\,dr$)

$$x^2+y^2=r^2$$

<u>Ex</u> $dA = r\, dr\, d\theta$

$dr$: Shoot, Slide; $d\theta$: Fix



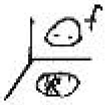

<u>Why use PCs?</u>

If $R$ is bounded by lines/rays, circular arcs, basic polar graphs, or

If the integrand has $x^2+y^2$, etc.

## 17.4: SURFACE AREA

$$S = \iint_R \sqrt{1 + [f_x(x,y)]^2 + [f_y(x,y)]^2}\, dA$$




Use PCs?

## 17.5: $\iiint$s

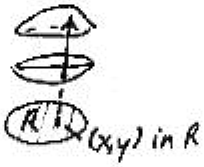
$$\iiint_Q f(x,y,z)\,\underbrace{dV}_{=dx\,dy\,dz \text{ (or permutation)}}$$

Ex

$$\iint_R \left[\int_?^{??} f(x,y,z)\,dz\right] dx\,dy$$

or

How do we shoot $z$ if we fix $x,y$?




What about $R$?

Can you find the projection of $Q$ on a coord. plane? (Here, $xy$-plane.)

Maybe look for 2 cylinders that "trap" space. Let $R$ be bounded by their traces in a coord. plane they're $\perp$ to.

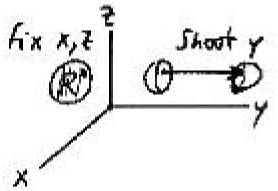
Use symmetry? PCs?

consider integrand, inner $\int$s

Can shift perspective

$$\iint_R \left[\int_{y=?}^{y=??} f(x,y,z)\,dy\right] dx\,dz$$

or



## 17.6: CENTER OF MASS

Mass $= m$

$$= \iint_R \underbrace{\delta(x,y)}_{\text{density}} \, dA$$

density
constant for a homogeneous lamina

Center of mass $= (\bar{x}, \bar{y})$

$$\bar{x} = \frac{\iint_R x \, \delta(x,y) \, dA}{m} \quad , \quad \bar{y} = \text{`` } \frac{y}{} \text{ ''}$$

Can use symmetry about $x=0$ (yz-plane), say, if

$\delta$ is even in $x$, and
$R$ is sym. about $x=0$.

$$m = 2 \iint_{\text{new } R} \quad , \text{ or } \quad \bar{x} = 0$$

Moments of inertia

$$I_y = \iint_R x^2 \, \delta(x,y) \, dA$$

$$I_x = \text{`` } y^2 \text{ ''}$$

$$I_0 = \text{`` } r^2 \text{ ''}$$

3D similar, except

$$I_z = \iiint_Q \underbrace{(x^2+y^2)}_{\text{sq. dist. from } z\text{-axis}} \delta(x,y,z) \, dV \text{ , etc.}$$

## 17.7: CYL. Cs

Cylindrical

$(\underbrace{r, \theta}_{\text{Polar}}, \underset{\text{Cartesian}}{z})$

Basic graphs

$dV = r\,dr\,d\theta\,dz$

Often

$$\iint_R \left[ \int_{z=?}^{z=??} \sim dz \right] r\,dr\,d\theta$$

or

Sketch!

Think: Polar

## 17.8: SCs

Spherical

$(\underset{[0,\infty)}{\rho}, \underset{[0,\pi]}{\phi}, \theta)$

Basic graphs

Sketch → Formulas

$r = \rho \sin\phi$

$\begin{cases} x = r\cos\theta = \rho\sin\phi\cos\theta \\ y = r\sin\theta = \rho\sin\phi\sin\theta \end{cases}$ y so sinful!

$z = \rho\cos\phi$

$\rho = \sqrt{x^2+y^2+z^2}$

$\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$

$\tan\theta = \frac{y}{x}$, watch Q!

$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

## 17.9: JACOBIANS, CHANGE OF VARIABLES

$$dA = \left\| \begin{matrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{matrix} \right\| du\, dv \quad \text{, where } \left. \begin{matrix} x = f(u,v), \\ y = g(u,v) \end{matrix} \right\} \begin{matrix} 1\text{-}1 \\ (u,v) \leftrightarrow (x,y) \end{matrix}$$

$$dV = \left\| \begin{matrix} \text{similar,} \\ 3\times 3 \end{matrix} \right\| du\, dv\, dw$$

$$\left. \begin{matrix} \text{Need new limits for } \iint, \iiint \\ \rightarrow u,v,w \text{ for integrand} \end{matrix} \right\} \begin{matrix} \text{Influence} \\ \text{choice} \\ \text{of sub} \end{matrix}$$