CH. 1: SYSTEMS OF LINEAR EQS.

1.1: INTRO

(A) Linear Eqs.

Standard Form

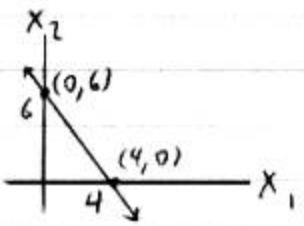
$$a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b$$

where "a_i"s are real coeffs.
"x_i"s are variables
"b" is a real #

Ex $3x_1 + 2x_2 = 12$

is a linear eq. in x_1 and x_2.

$a_1 = 3$
$a_2 = 2$
$b = 12$



Every pt. represents a solution.
∞ many sol'ns making up the solution set.

Before you'd say big deal - it's a line (Cartesian)

(B) Parametric Representation of a Solution Set (Sol'n)

Describe the sol'n set.

Ex Solve $3x_1 + 2x_2 = 12$

Solve for x_1 (say)

$$3x_1 = -2x_2 + 12$$

$$x_1 = -\frac{2}{3}x_2 + 4$$

x_1: depends on x_2 — x_2: free variable (plug in)

Let $x_2 = t$ ("parameter")

Let's say I have a (boring) friend, and he wants to know some sol'ns to this eq. Here's a recipe for producing sol'ns. Plug in the into t. 3, -5, π

Sol'n set:

$$x_1 = -\frac{2}{3}t + 4$$
$$x_2 = t$$

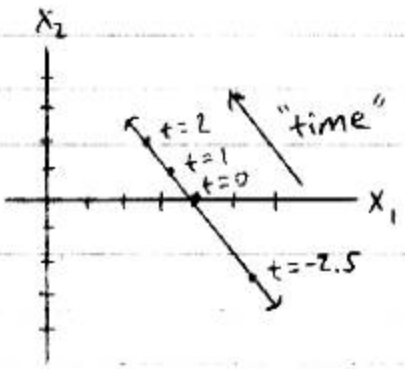
t is any real #

Table of particular sol'ns

Do x_2 1st.

what does t often stand for?

t	x_1	x_2	
0	4	0	→ (4, 0)
1	$3\frac{1}{3}$	1	
2	$2\frac{2}{3}$	2	
-2.5	$5\frac{2}{3}$	-2.5	
⋮			



Ex Solve $2x_1 + 3x_2 - 4x_3 = 1$

Solve for x_1

$$2x_1 = -3x_2 + 4x_3 + 1$$

$$x_1 = -\tfrac{3}{2}x_2 + 2x_3 + \tfrac{1}{2}$$

2 free vars.

Let $x_2 = t$

$x_3 = u$

Sol'n set:

$$x_1 = -\tfrac{3}{2}t + 2u + \tfrac{1}{2}$$
$$x_2 = t$$
$$x_3 = u$$

t, u are any real #s

Table

Do x_2, x_3 1st.

t	u	x_1	x_2	x_3	
0	0	$\frac{1}{2}$	0	0	①
1	0	-1	1	0	②
1	1	1	1	1	③
-3	$5\frac{1}{2}$	16	-3	$5\frac{1}{2}$	

As you vary values for t, u, you sweep out an entire plane of sols.

As you vary t from 0 to 1 you sweep out pts on this line seg.

In $x_1x_2x_3$-space plane of sol'ns

© Solving Systems of Linear Eqs.

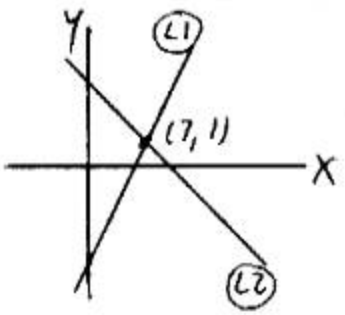
Ex Solve $\begin{cases} 3x - 2y = 19 & (L1) \\ x + y = 8 & (L2) \end{cases}$

A solution to this system solves <u>all</u> the eqs.

Method 1 (Graphing)

This line gives all sols to this eq...

If the soln is $(\frac{2}{8}, \frac{17}{12})$



An intersection point is a solution. (2D)

Method 2 (Substitution)

We solve for one of the vars in one of the eqs.

$$\begin{cases} 3x - 2y = 19 \\ x + y = 8 \end{cases} \Rightarrow y = 8 - x$$

$$3x - 2(8 - x) = 19$$

$$\vdots$$

$$x = 7$$

Am I done?

$$y = 8 - x = 8 - 7 = 1$$

$$\begin{cases} x = 7 \\ y = 1 \end{cases} \quad \underline{\text{or}} \quad \{(7, 1)\}$$

sol'n set consisting of 1 sol'n an ordered pair

Method 3 (Addition Method)

$$\begin{cases} 3x - 2y = 19 \\ x + y = 8 \end{cases} \quad \leftarrow \cdot 2$$

When we add equals to equals, we get equals.

$$\begin{cases} 3x - 2y = 19 \\ 2x + 2y = 16 \end{cases} \quad \downarrow \text{Add eqs.}$$

$$5x = 35$$

$$x = 7$$

$$7 + y = 8$$

$$y = 1$$

$\{(7, 1)\}$

More methods later!

① Types of Solution Sets

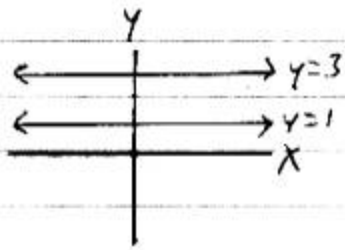
For systems in $y+x$

# of sol'ns = # of intersection pts.

When do 2 lines in the xy-plane never intersect?

Ex (0 sol'ns)

$$\begin{cases} y = 1 \\ y = 3 \end{cases}$$



Different parallel (||) lines never intersect.

Sol'n set = ϕ (empty/null set)

We can't reconcile the eqs.

The system is <u>inconsistent</u>.

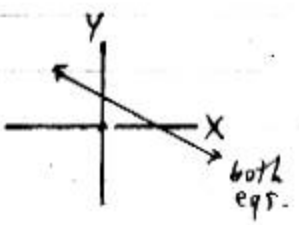
Ex (1 sol'n)

not ||

How can 2 lines in the xy-plane intersect each other ∞ many times?

Ex (∞ many sol'ns)

$$\begin{cases} x + 2y = 5 \\ 2x + 4y = 10 \end{cases}$$ ↩ same stupid line!



The equations are <u>dependent</u>.

To write soln set:

Parametric rep.

$x + 2y = 5$

$x = 5 - 2y$ ← free

Let $y = t$

$x = 5 - 2t$

$y = t$

t is any real #

Summary

# sol'ns	Ex	System is	Eqs. are (96, not 254)
0	⇉	inconsistent	independent
1	⤭	consistent	independent
∞ many	↕ (both)	consistent	dependent

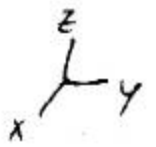
↑

Only possibs. for systems of linear eqs.

(E) Systems of 3 Linear Eqs. in 3 Variables

Ex $\begin{cases} x + 2y - z = -3 \\ 2x - y + 3z = 4 \\ z = 3 \end{cases}$ → plane

(The graph of each eq. is a plane)

in 

1 sol'n: $(-2, 1, 3)$ (x, y, z)

All 3 planes intersect here.
A solution must lie on all 3 planes.

In a hospital Descartes sees a fly on a wall

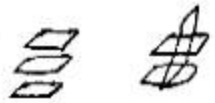
Sick, nothing better to do, traces its movement relative to the walls.

Idea 2 planes can intersect in a line "L"

corner of a room

A 3rd plane can intersect L in 1 point.

What if only 2 planes are ||?

Exs (0 sol'ns)  (no point on all 3)

Exs (∞ many sol'ns)   (same stupid plane)

1∞ line of sol'ns in 3-space

2∞ plane of sol'ns in 3-space