1.2: MATRIX METHODS FOR SOLVING SYSTEMS

Ⓐ Matrices

A <u>matrix</u> is a box of #s.

<u>Ex</u> A $\underbrace{2 \overset{}{\underset{\overset{\uparrow}{\text{“by”}}}{\times}} 3}_{\underline{\text{size}}}$ matrix:

$$\begin{array}{rc}
\begin{array}{r}2 \rightarrow\\ \underline{\text{rows}} \rightarrow\end{array} & \begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 5\end{bmatrix}\\
 & \begin{array}{ccc}\uparrow & \uparrow & \uparrow\end{array}\\
 & 3\ \underline{\text{columns}}
\end{array}$$

has 6 <u>entries</u>/elements

Form for an $m \times n$ matrix:

$$\begin{array}{rc}
\begin{array}{r}\rightarrow\\ \overset{m}{\text{rows}} \rightarrow\\ \\ \rightarrow\end{array} & \begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & & & \vdots\\ a_{m1} & a_{m2} & \cdots & a_{mn}\end{bmatrix}\\
 & \begin{array}{cccc}\uparrow & \uparrow & \quad & \uparrow\end{array}\\
 & n\ \text{columns}
\end{array}$$

has mn entries: the "$\underset{\text{row}\nearrow\ \nwarrow\text{col.}}{a_{ij}}$"'s square $\qquad m = n$ "order" $\qquad {}_{n}^{1}\begin{bmatrix} ^{1} & & ^{n} \\ & & \end{bmatrix}$

Ⓑ Writing the Augmented Matrix for a System

① Write the eqs. in standard form

Ex
$$\begin{cases} \boxed{3x + 2y - z = 0} \ ✓ \\ \boxed{-2x + y = 3} \ ✓ \quad \text{(OK that coeff. of } x \text{ is "}-\text{")} \\ y = 4z - 5 \rightarrow \boxed{y - 4z = -5} \ ✓ \end{cases}$$

② Line up like terms (in columns)

$$\begin{cases} 3x + 2y - z = 0 \\ -2x + y = 3 \\ y - 4z = -5 \end{cases}$$

$$\left(\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ x & y & z \\ \text{terms} & & \end{array} \right)$$

③ (Optional). Write in $0, \pm 1$ coeffs.

$$\left(\begin{array}{ccc} +y & \rightarrow & +1y \\ -z & \rightarrow & -1z \\ \text{no } x \text{ term} & \rightarrow & 0x \end{array} \right)$$

$$\begin{cases} 3x + 2y - 1z = 0 \\ -2x + 1y + 0z = 3 \\ 0x + 1y - 4z = -5 \end{cases}$$

④ Write the <u>augmented matrix</u>

$$\begin{array}{ccc|c} & \text{coeffs. of} & & \text{Right-hand} \\ \underline{x} & \underline{y} & \underline{z} & \underline{\text{side. (RHS)}} \\ 3 & 2 & -1 & 0 \\ -2 & 1 & 0 & 3 \\ 0 & 1 & -4 & -5 \end{array}$$

coefficient matrix

(book omits)

Even though it's tempting to do it by columns, you may be better off doing it by rows. When you read the eqs., you're less likely to omit the ±1s. Still, lining up like terms a good idea.

skip

Ex

$$\begin{cases} 3x - y = -4 \\ 5x = 7 \end{cases}$$

$$\begin{cases} 3x - 1y = -4 \\ 5x + 0y = 7 \end{cases}$$

$$\left[\begin{array}{cc|c} 3 & -1 & -4 \\ 5 & 0 & 7 \end{array}\right]$$

Ⓒ Elementary Row Operations (EROs)

Equivalent equations have the same sol'n set.

Ex $2x-1=5$
$2x=6$
$x=3$

↓ sequence of equiv. eqs.

← an eq. whose sol'n is obvious

Equivalent systems have the same sol'n set.

EROs: legal chess moves

EROs allow us to write a sequence of equivalent systems until we get one whose sol'n is easy to find.

EROs:

① Row Interchange

If we have a sol'n to this system, What does it do?

Ex $\left\{\begin{array}{l} 3x-y=1 \\ x+y=4 \end{array}\right. \rightarrow \left\{\begin{array}{l} x+y=4 \\ 3x-y=1 \end{array}\right.$ (same sol'n set)

$$\begin{array}{l} R_1\rightarrow \\ R_2\rightarrow \end{array}\left[\begin{array}{cc|c} 3 & -1 & 1 \\ 1 & 1 & 4 \end{array}\right] \underset{R_1\leftrightarrow R_2}{\sim} \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 3 & -1 & 1 \end{array}\right]$$

(We can rewrite the rows in any order.)

② Row Rescaling

We can $\overset{\text{mult.}}{\cdot}$ (or ÷) through a row
by any (non-0) #.

Ex $\begin{cases} \frac{1}{2}x + \frac{1}{2}y = 3 \\ y = 4 \end{cases} \left(\rightarrow \begin{cases} x + y = 6 \\ y = 4 \end{cases} \right)$

$$\cdot 2 \rightarrow \left[\begin{array}{cc|c} \frac{1}{2} & \frac{1}{2} & 3 \\ 0 & 1 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 1 & 4 \end{array}\right]$$

$$\boxed{\underset{(\text{old})}{2 \cdot R_1} \rightarrow \underset{(\text{new})}{R_1}}$$

③ Row Replacement

We can add a multiple of one row
to another row. (Idea: Addition Method.)

It turns out
we want to
turn the "-2"
into a "0"

Ex

$$\begin{cases} x + 3y = 3 \\ -2x + 5y = 16 \end{cases} \leftarrow (\cdot 2) \quad \text{(omit)}$$

$$\begin{array}{r} 2x + 6y = 6 \\ -2x + 5y = 16 \\ \hline 11y = 22 \end{array} \quad \begin{array}{l} \leftarrow \text{revert to original} \\ \text{can replace} \end{array}$$

(hybrid eq.)

$$\begin{cases} x + 3y = 3 \\ 11y = 22 \end{cases}$$

$$\begin{array}{c} R_1 \\ R_2 \end{array} \left[\begin{array}{cc|c} 1 & 3 & 3 \\ \underset{\text{want } 0}{\boxed{-2}} & 5 & 16 \end{array}\right]$$

$$\boxed{\underset{(\text{old})}{R_2} + 2 \cdot R_1 \rightarrow \underset{(\text{new})}{R_2}}$$

$$\begin{array}{rcc|c} R_2: & -2 & 5 & 16 \\ + \; 2 \cdot R_1: & 2 & 6 & 6 \\ \hline \text{new } R_2: & 0 & 11 & 22 \end{array} \quad \downarrow \text{Add}$$

$$\begin{array}{c} \text{Same } R_1 \\ \text{New } R_2 \end{array} \left[\begin{array}{cc|c} 1 & 3 & 3 \\ 0 & 11 & 22 \end{array}\right] \leftarrow \div 11$$

$$\left[\begin{array}{cc|c} 1 & 3 & 3 \\ 0 & 1 & 2 \end{array}\right] \quad \text{Good form!}$$

Only ≈40%
have seen

① Gaussian Elimination (Easy Exs.)

Meyer p.1
Chinese 200 BC
→ Europe
Gauss popularized
it.

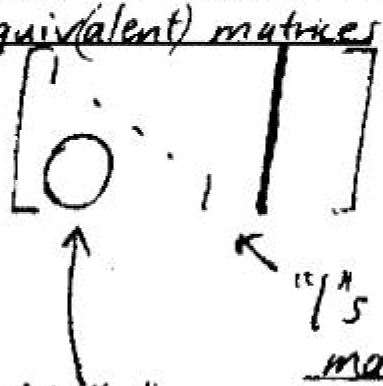
Method for solving systems.

Steps

① Write the augmented matrix.

② Use EROs to write a sequence of row-equiv(alent) matrices until you get the form

What's
Checkmate?



Top of a
jigsaw puzzle
box.

All "0"s
below

"1"s along the
main diagonal

(if coeff. matrix
is square)

Strategy: "Correct" the columns from left-to-right.

You don't want
to undo the
good work
you've done.

③ Write the new system.

④ Use back-sub(stitution) to find the solution.

Before you
got your
dirty hands
all over it...

Can ✓ in original system.

Solve

Ex Lial 24

$$\begin{cases}3x-5y=7\\ x-y=1\end{cases}$$

$$\left(\begin{cases}3x-5y=7\\ 1x-1y=1\end{cases}\quad\text{(optional)}\right)$$

Augmented) Matrix

want "1"

$$\left[\begin{array}{cc|c}\enclose{circle}{3} & -5 & 7\\ 1 & -1 & 1\end{array}\right] \leftarrow \text{If we } \div 3\text{, fractions!}$$

Let's $R_1 \leftrightarrow R_2$ (Write steps - in case you $\underset{\text{back}}{\checkmark}$ later.)

$$\left[\begin{array}{cc|c}\overset{\ddot\smile}{\enclose{circle}{1}} & -1 & 1\\ \enclose{circle}{3} & -5 & 7\end{array}\right] (\leftarrow \text{can't } \cdot 0)$$

want "0"
(elimination)

$$\left(\text{Let's:}\quad \underset{\text{(old)}}{R_2} - 3\cdot R_1 \to \underset{\text{(new)}}{R_2} \quad R_2 \overset{\text{or}}{+} (-3)\cdot R_1\right)$$

$$\begin{array}{rrr|r} R_2: & 3 & -5 & 7\\ +(-3)\cdot R_1: & -3 & 3 & -3\\ \hline \text{new } R_2: & 0 & -2 & 4\end{array}\ \downarrow\text{Add}$$

Don't want to use this row - we'll disturb "0"

$$\left[\begin{array}{cc|c}1 & -1 & 1\\ 0 & \enclose{circle}{-2} & 4\end{array}\right] \leftarrow \div(-2) \text{ or } \cdot\left(-\tfrac{1}{2}\right)$$

want "1" here

$$\begin{array}{c} \begin{array}{cc} x & \quad y \end{array} \\ \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 1 & -2 \end{array}\right] \end{array}$$

⇒ System

$$\left(\begin{cases} 1x - 1y = 1 \\ 0x + 1y = -2 \end{cases} \quad \begin{pmatrix} \text{can} \\ \text{skip} \end{pmatrix} \right)$$

$$\begin{cases} x - y = 1 \\ \quad\;\; y = -2 \end{cases} \uparrow$$

Back-Sub

$$\boxed{y = -2}$$

(You)

$$x - y = 1$$
$$x - (-2) = 1$$
$$x + 2 = 1$$
$$\boxed{x = -1}$$

What's my sol'n?

$$\boxed{\{(\overset{x}{-1}, \overset{y}{-2})\}}$$

up to 23, except 7, 9, 11, 21

Can ✓ (in original system.)

Ex Solve
$$\begin{cases} 3x_1 + 2x_2 + x_3 = 8 \\ 6x_1 - x_2 + 3x_3 = 18 \\ -9x_1 + x_2 - x_3 = -20 \end{cases}$$

Like Tussy #32

$$\left(\begin{cases} 3x_1 + 2x_2 + 1x_3 = 8 \\ 6x_1 - 1x_2 + 3x_3 = 18 \\ -9x_1 + 1x_2 - 1x_3 = -20 \end{cases} \quad \text{(optional)}\right)$$

6, -9 are mults. of 3

Use →; want "0"

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 8 \\ 6 & -1 & 3 & 18 \\ -9 & 1 & -1 & -20 \end{array}\right] \quad (\leftarrow \text{If we } \div 3\text{, fractions early!})$$

↑ Want $\begin{matrix}1\\0\\0\end{matrix}$

$$\begin{array}{r|rrr|r} R_2: & 6 & -1 & 3 & 18 \\ +\ (-2)\cdot R_1: & -6 & -4 & -2 & -16 \\ \hline \text{new } R_2: & 0 & -5 & 1 & 2 \end{array} \quad \downarrow \text{Add}$$

$$\begin{array}{r|rrr|r} R_3: & -9 & 1 & -1 & -20 \\ +\ 3\cdot R_1: & 9 & 6 & 3 & 24 \\ \hline \text{new } R_3: & 0 & 7 & 2 & 4 \end{array}$$

Let's assume you're not afraid of fracs for now. Can ÷7 ÷5

$$\begin{array}{c} \text{want "1"} \\ \left[\begin{array}{ccc|c} \boxed{3} & 2 & 1 & 8 \\ 0 & \boxed{-5} & 1 & 2 \\ 0 & 7 & 2 & 4 \end{array}\right] \begin{array}{l} \leftarrow \div 3 \text{ (maybe later)} \\ \leftarrow \div(-5) \\ \end{array} \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 8 \\ 0 & -35 & 7 & 14 \\ 0 & 35 & 10 & 20 \end{array}\right] R_3+R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 8 \\ 0 & -35 & 7 & 14 \\ 0 & 0 & 17 & 34 \end{array}\right] \begin{array}{l} \\ \leftarrow \div 7 \\ \leftarrow \div 17 \end{array}$$

↖ Other books: OK

→ System avoids fractions

$$\left[\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & \frac{8}{3} \\ 0 & 1 & -\frac{1}{5} & -\frac{2}{5} \\ 0 & \boxed{7} & 2 & 4 \end{array}\right] \leftarrow \text{"in book"}$$

want "0"

$$\begin{array}{rcccc|c} & R_3: & 0 & 7 & 2 & 4 \\ & (-7)\cdot R_2: & 0 & -7 & \frac{7}{5} & \frac{14}{5} \\ \hline & \text{new } R_3: & 0 & 0 & \frac{17}{5} & \frac{34}{5} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{2}{3} & \frac{1}{3} & \frac{8}{3} \\ 0 & 1 & -\frac{1}{5} & -\frac{2}{5} \\ 0 & 0 & \boxed{\frac{17}{5}} & \frac{34}{5} \end{array}\right] \leftarrow \div\left(\frac{17}{5}\right)$$

want "1"

(Some books allow you to stop quicker here. Key: 0s below → main diag)

Notice $\frac{34}{5}$ is twice $\frac{17}{5}$

I demand that you go all the way!

$$\left[\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ 1 & \frac{2}{3} & \frac{1}{3} & \frac{8}{3} \\ 0 & 1 & -\frac{1}{5} & -\frac{2}{5} \\ 0 & 0 & 1 & 2 \end{array}\right]$$

⇒ System

$$\left\{\begin{array}{rcl} x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_3 & = & \frac{8}{3} \\ x_2 - \frac{1}{5}x_3 & = & -\frac{2}{5} \\ x_3 & = & 2 \end{array}\right. \quad \Big\uparrow$$

Back-Sub

$$\boxed{x_3 = 2}$$

$$\begin{aligned} x_2 - \tfrac{1}{5}x_3 &= -\tfrac{2}{5} \\ x_2 - \tfrac{1}{5}(2) &= -\tfrac{2}{5} \\ x_2 - \tfrac{2}{5} &= -\tfrac{2}{5} \\ \boxed{x_2 = 0} \end{aligned}$$

$$\begin{aligned} x_1 + \tfrac{2}{3}x_2 + \tfrac{1}{3}x_3 &= \tfrac{8}{3} \\ x_1 + \cancel{\tfrac{2}{3}(0)} + \tfrac{1}{3}(2) &= \tfrac{8}{3} \\ x_1 + \tfrac{2}{3} &= \tfrac{8}{3} \\ x_1 &= \tfrac{6}{3} \\ \boxed{x_1 = 2} \end{aligned}$$

Watch order here - lucky - palindrome

$$\boxed{\{(\overset{x_1}{2},\overset{x_2}{0},\overset{x_3}{2})\}}$$

Can ✓

Up to 25, except 7, 9, 11, 21

See $\boxed{\text{Ex 3}}$ (p.16), $\boxed{\text{Ex 5}}$ (p.18) - for more practice.

Ⓔ When Does a System Have No Solution?

If we *ever* get a row

$$\underbrace{0 \quad 0 \;\cdots\; 0}_{\substack{\text{(all "0"'s on)} \\ \text{the coeff. side}}} \;\Big|\; \underbrace{\text{a non-0 \#}}_{\text{RHS}}$$

$\boxed{\text{STOP!}}$ Sol'n set $= \phi$
System "inconsistent"

Ex $\begin{cases} x + y = 1 \\ x + y = 4 \end{cases}$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 1 & 4 \end{array}\right]$$

Tech, $R_2 + (-1)R_1$

$$R_2 - R_1 \to R_2$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 3 \end{array}\right] \leftarrow 0=3 \text{ can never be satisfied}$$

$\textcircled{\phi}$

Up to 25
except
7, 9, 11

See $\textcircled{\text{Ex 6}}$ (p.20)

1.2.13

Ⓕ Row-Echelon Form for a Matrix

We aimed for

$$\left[\begin{array}{ccc|c} 1 & & & \\ & 1 & & \\ 0 & & \ddots \; 1 & \end{array}\right]$$

special case of

What if we can't get this? Ex [not square |]

isn't even square?)

In general, we aim for row-echelon form.

Properties

① If there are any "all-0" rows, they must be at the bottom.

Aside from "all-0" rows,

② Every row must have a "1" (leading 1) as its leftmost non-0 entry.

(③ The "leading 1" of a row must always be to the right of the "leading 1"s of all higher rows. (1 ↳ 1 ↳ 1))

They go down and to the right. JFK

③ The "leading 1"s go down and to the right. (① ↳ ① ↳ ①)

1.2.14

is this matrix in row-ech form?

Ex
$$\left[\begin{array}{ccccc|c} 0 & \textcircled{1} & 3 & 1 & 4 & 7 \\ 0 & 0 & 0 & \textcircled{1} & 9 & 4 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

not leading

not in book

("Leading 1"s are in the pivot positions

Ex Solve
$$\begin{cases} 2x-3y=21 \\ 4x-6y=42 \\ 2x-2y=16 \end{cases}$$

$$\begin{array}{r} \\ R_2-2R_1 \to 0 \\ R_3-R_1 \to 0 \end{array} \left[\begin{array}{cc|c} 2 & -3 & 21 \\ 4 & -6 & 42 \\ 2 & -2 & 16 \end{array}\right]$$

What do we get? Walk around

or ÷2 1st

$$\left[\begin{array}{cc|c} 2 & -3 & 21 \\ 0 & 0 & 0 \\ 0 & 1 & -5 \end{array}\right] \circlearrowleft$$

$$1 \leftarrow \left[\begin{array}{cc|c} 2 & -3 & 21 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{array}\right] \leftarrow \div 2$$

$$\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & \frac{21}{2} \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{array}\right]$$

→System

$$\begin{cases} x - \frac{3}{2}y = \frac{21}{2} \\ \quad\;\; y = -5 \\ \quad\;\; \cancel{0 = 0} \end{cases}$$

(any sol'n of this automatically satisfies "0=0")

← Can drop

Back-Sub

$$\boxed{y = -5}$$

$$x - \frac{3}{2}y = \frac{21}{2}$$
$$x - \frac{3}{2}(-5) = \frac{21}{2}$$
$$x + \frac{15}{2} = \frac{21}{2}$$
$$\left(x = \frac{6}{2}\right)$$
$$\boxed{x = 3}$$

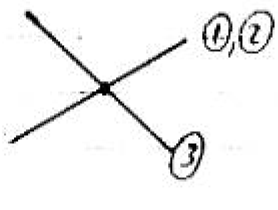
$$\boxed{\{(\overset{x}{3}, \overset{y}{-5})\}}$$

Can ✓

Note

$$\begin{cases} 2x - 3y = 21 \\ 4x - 6y = 42 \\ 2x - 2y = 16 \end{cases}$$

same stupid line!



1.2.16

⑥ Reduced Row-Echelon (RRE) Form

Recite ①-③

Row-echelon form **and**

0s directly above, below

Property ④: Each "leading 1" has all "0"'s elsewhere in its column.

Old Ex Solve $\begin{cases} 2x - 3y = 21 \\ 4x - 6y = 42 \\ 2x - 2y = 16 \end{cases}$

using Gauss-Jordan elimination
(→ RRE form).

Augmented Matrix

$$\left[\begin{array}{cc|c} 2 & -3 & 21 \\ 4 & -6 & 42 \\ 2 & -2 & 16 \end{array}\right]$$

$$\downarrow$$

Which # must → 0

We need to eliminate up from the leading 1s.

Old Ex

$$\left[\begin{array}{cc|c} 1 & \overset{\text{Need "0" for RRE form}}{\boxed{-\frac{3}{2}}} & \frac{21}{2} \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{array}\right] \quad \text{Row-Echelon Form}$$

In Gauss-Jordan elimination, we → RRE form

$$\begin{array}{rccc} R_1: & 1 & -\frac{3}{2} & \frac{21}{2} \\ +\frac{3}{2}R_2: & 0 & \frac{3}{2} & -\frac{15}{2} \\ \hline \text{new } R_1: & 1 & 0 & 3 \end{array}$$

"Work" moves from Back-Sub stage to here

1.2.17

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{array}\right]$$ <u>RRE form</u>

$$\begin{cases} x = 3 \\ y = -5 \end{cases}$$ (Sol'n pops out!)

MATLAB
If you're working
w/ a friend

<u>Note</u> Every matrix has a <u>unique</u> RRE form.

<u>Ex 7 (p. 21)</u>

Book:

Row-echelon form → RRE form

$$\left[\begin{array}{ccc|c} 1 & \overset{\text{Book: 1st} \nearrow 0}{\enclose{circle}{-2}} & \overset{\nearrow 0}{\enclose{circle}{3}} & 9 \\ 0 & 1 & \enclose{circle}{3} \nearrow 0 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Faster: "Clear up" columns from right-to-left.

We like to take
advantage of 0.
He's easy to
push around.
Take his lunch
money.

(You get to take advantage of "0"s along the way.)

Ⓗ Systems with ∞ Many Sol'ns

Col of 0s
see Lf-4+

Ex Solve $\begin{cases} x_1 - 2x_2 + x_3 + 5x_4 = 3 \\ 2x_1 - 4x_2 + x_3 + 7x_4 = 5 \end{cases}$

Augmented Matrix

$$R_2 - 2R_1 \rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 1 & 5 & 3 \\ 2 & -4 & 1 & 7 & 5 \end{array}\right]$$

$$\cdot(-1) \rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 1 & 5 & 3 \\ 0 & 0 & -1 & -3 & -1 \end{array}\right]$$

$$R_1 - R_2 \rightarrow \left[\begin{array}{cccc|c} \textcircled{1} & -2 & 1 & 5 & 3 \\ 0 & 0 & \textcircled{1} & 3 & 1 \\ \textcircled{x_1} & \boxed{x_2} & \textcircled{x_3} & \boxed{x_4} & \end{array}\right]$$

Row-Echelon Form

Basic vars

Free vars. correspond to columns in the coeff. matrix that do not have a (leading 1).
(Here, x_2 and x_4.)

If there are free vars and the system is consistent (no row $[0\ 0 \cdots 0 | \neq 0]$),
$\Rightarrow$ ∞ many sol'ns.

You must → RRE form to describe the sol'n set.

1.2.19

RRE Form

$$\left[\begin{array}{cccc|c} \textcircled{1} & -2 & 0 & 2 & 2 \\ \underset{x_1}{0} & \underset{\underset{\text{(free)}}{x_2}}{0} & \underset{x_3}{\textcircled{1}} & \underset{\underset{\text{(free)}}{x_4}}{3} & 1 \end{array}\right].$$

→ System

$$\begin{cases} x_1 - 2x_2 \qquad + 2x_4 = 2 \\ \qquad\qquad x_3 + 3x_4 = 1 \end{cases}$$

Move all free vars. to the right side.

$$\begin{cases} x_1 = 2 + 2x_2 - 2x_4 \\ x_3 = 1 \qquad\quad - 3x_4 \end{cases}$$

Parametrization

Let $x_2 = t$, $x_4 = u$ (e.g. Book uses s, t)

Sol'n Set in Parametric Form

$$\boxed{\begin{aligned} x_1 &= 2 + 2t - 2u \\ x_2 &= \quad\;\; t \\ x_3 &= 1 \qquad\; - 3u \\ x_4 &= \qquad\qquad u \end{aligned}}$$

t, u are any real #s.

Some Sol'ns

t	u	x_1	x_2	x_3	x_4
0	0	2	0	1	0
-4	7	-20	-4	-20	7
⋮					

$\underset{\substack{\uparrow \\ \text{2 free vars}}}{2}$D plane of sol'ns in $\underset{\substack{\uparrow \\ \text{4 vars}}}{4}$D space.

See Ex 8 (p. 22)

1.2.21

① Homogeneous Systems of Linear Eqs.

(have augmented matrices that have all "0"'s in RHS.)

$$\left[\begin{array}{c|c} & \text{RHS} \\ \qquad\qquad & 0 \\ & 0 \\ & \vdots \\ & 0 \end{array}\right]$$

Ex $\begin{cases} x + y - z = 0 \\ 2x + y - 3z = 0 \end{cases}$ $\boxed{\text{Consistent}}$

We (automatically) know $(\overset{x}{0}, \overset{y}{0}, \overset{z}{0})$ is a sol'n (the trivial sol'n).

(Any such system is consistent.)

If more vars. than eqs., then

$$\left[\begin{array}{c|c} \text{"fat"} & 0 \\ \left(\begin{array}{c}\text{coeff.} \\ \text{matrix}\end{array}\right) & \vdots \\ & 0 \end{array}\right]$$

we're going to have what kind of vars?

There will be free vars.

∞ many sol'ns.

Note

$$\left[\begin{array}{c|c} \text{square} & \begin{matrix}0\\ \vdots\\ 0\end{matrix}\end{array}\right] \qquad \left[\begin{array}{c|c} \text{skinny} & \begin{matrix}0\\ \vdots\\ 0\end{matrix}\end{array}\right]$$

can have 1 or ∞ many sol'ns.

Use ≥2 vars.

Ex (You)

Homog. system w/more eqs. than vars. that has ∞ many sol'ns

Write →
They got it!

$$\left\{\begin{array}{l} x + y = 0 \\ 2x + 2y = 0 \\ 3x + 3y = 0 \end{array}\right. \begin{array}{l} \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{can drop} \end{array} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{same stupid line}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} \textcircled{1} & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{cc} x & \begin{matrix} y \\ \text{(free)} \end{matrix} \end{array}$$

Too easy

Ex 1 sol'n

$$\left\{\begin{array}{r} x \qquad = 0 \\ y = 0 \\ 2y = 0 \end{array}\right. \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\{(0,0)\}$

1.2.23

① Combos

Go col by col

A skinny can do whatever a square can do

		# sol'ns	
	0	1	∞ many
$\left[\begin{array}{c\|c}\text{square} & \ \end{array}\right]$	$\left[\begin{array}{cc\|c}1 & 2 & 7\\0 & 0 & 1\end{array}\right]$	$\left[\begin{array}{cc\|c}1 & 0 & 3\\0 & 1 & 4\end{array}\right]$	$\left[\begin{array}{cc\|c}1 & 1 & 3\\0 & 0 & 0\end{array}\right]$
$\left[\begin{array}{c\|c}\text{skinny} & \ \end{array}\right]$	$\left[\begin{array}{cc\|c}1 & 2 & 7\\0 & 0 & 1\\0 & 0 & 0\end{array}\right]$	$\left[\begin{array}{cc\|c}1 & 0 & 3\\0 & 1 & 4\\0 & 0 & 0\end{array}\right]$	$\left[\begin{array}{cc\|c}1 & 1 & 3\\0 & 0 & 0\\0 & 0 & 0\end{array}\right]$
$\left[\begin{array}{c\|c}\text{fat} & \ \end{array}\right]$	$\left[\begin{array}{ccc\|c}1 & 2 & 3 & 7\\0 & 0 & 0 & 1\end{array}\right]$	NO!	$\left[\begin{array}{ccc\|c}1 & 0 & 1 & 3\\0 & 1 & 0 & 4\end{array}\right]$

(Is there a relationship between matrix size
and # sols.? Not really, except (NO!))
(Other than that, anything goes!)