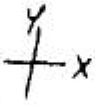
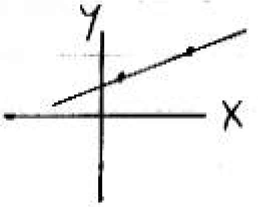
1.3: APPLICATIONS

Ⓐ Polynomial Curve Fitting in 

Why diff? Graph can't fail VLT

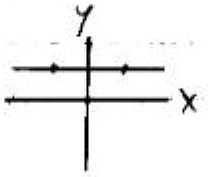
n points w/different x-coords.:
determine a polynomial function of degree $\leq n-1$.

2 pts.



$p(x) = a_0 + a_1 x$ (deg. 1)

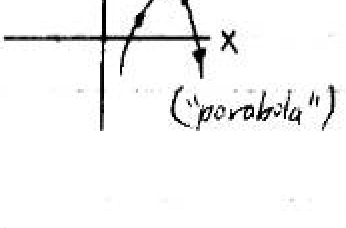
Special Case



$p(x) = a_0$ (deg. 0)

Here, $a_1 = 0$.

3 pts.



$p(x) = a_0 + a_1 x + a_2 x^2$ (deg. 2)

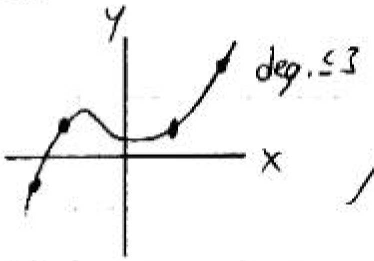
Special Cases

deg. 1

deg. 0

There are higher-deg. funcs. that will do this..

4 pts.



n pts.

$$p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1} \text{ will do}$$

"a_i"'s can be 0

deg $\leq n-1$

Ex Find a polynomial) function where graph passes through $(-1, 2)$, $(0,-1)$, and $(3,2)$.

$$\underbrace{p(x)}_{\text{or } y} = a_0 + a_1 x + a_2 x^2 \quad \text{will do}$$

Plug in

$(\overset{x}{-1}, \overset{y}{2})$ $\quad 2 = a_0 + a_1(-1) + a_2(-1)^2$

$(0,-1)$ $\quad -1 = a_0 + a_1(0) + a_2(0)^2$

$(3, 2)$ $\quad 2 = a_0 + a_1(3) + a_2(3)^2$

In practice plug $a_0 = -1$ into other ⇒ system of 2 eqs.

$$\text{Solve} \begin{cases} a_0 - a_1 + a_2 = 2 \\ a_0 = -1 \\ a_0 + 3a_1 + 9a_2 = 2 \end{cases}$$

ae (Golub ??) must be consistent — is always! $1, x, \ldots, x^{n-1}$ basis for P_n
What's this col? p120 Num. Recipes

Vandermonde matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 1 & 0 & 0 & -1 \\ 1 & 3 & 9 & 2 \end{array}\right]$$

$\uparrow$ "1"s $\quad$ $\uparrow$ "x"s $\quad$ $\uparrow$ "x^2"s $\quad$ $\uparrow$ "y"s (Shortcut (4 pts. → cubic → "x^3"s))

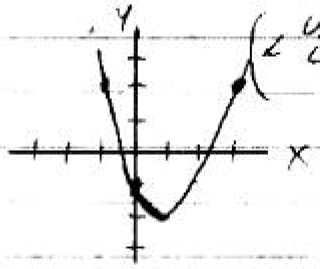
Solution: $\begin{cases} a_0 = -1 \\ a_1 = -2 \\ a_2 = 1 \end{cases}$

$$p(x) = a_0 + a_1 x + a_2 x^2$$

$$p(x) = -1 - 2x + x^2$$

or

$$x^2 - 2x - 1$$

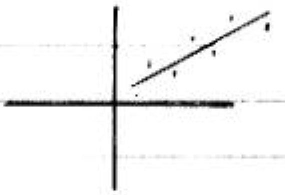


(use computer)

Did you find a quintic?

In stats

you don't want your model to be too dependent on noise

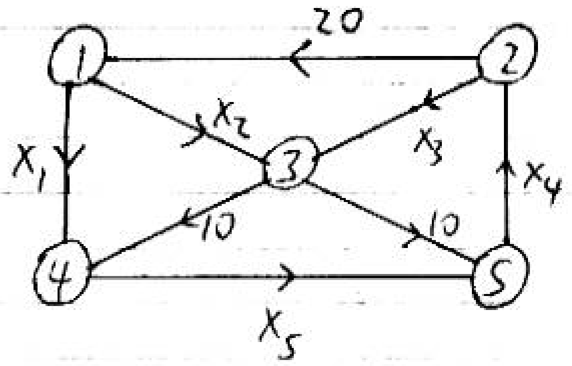


Best-fit line
may summarize data better

(B) Network Analysis

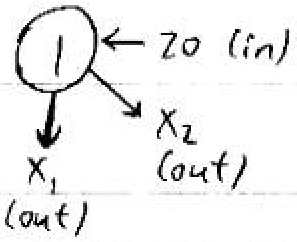
Read Ex. 5 on pp. 31-32

5 junctions (circles)

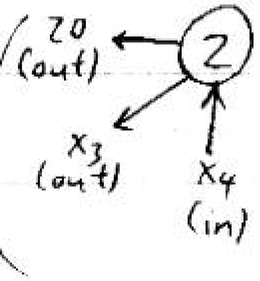


"balancing act"

(At each junction,) flow in = flow out



$$20 = x_1 + x_2$$



$$x_4 = x_3 + 20$$

etc.

We get a system of 5 linear eqs. Maybe ∞ sol'ns.

May require all $x_i \geq 0$, integer.