CH 2: MATRICES

2.1: MATRIX OPERATIONS

Ⓐ Notation

$A, B, C, \ldots$ usually denote matrices.

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & & \\ & & \ddots & \\ & & & a_{mn} \end{bmatrix}$$

(i.e., A consists of entries "a_{ij}" $A = \begin{bmatrix} a_{11} & a_{12} & \\ & \ddots & a_{mn} \end{bmatrix}$)

$$B = [b_{ij}]$$

row vector: (a matrix w/only 1 row.)

Ex $[\,1 \quad 0 \quad \sqrt{2}\,]$

components

column vector: (a matrix w/only 1 column.)

Denoted by $\vec{a}, \vec{b}, \ldots$

book: boldfaced

$\left(\underline{Ex} \quad \vec{a} = \begin{bmatrix} 5 \\ -1 \\ 3/2 \end{bmatrix} \right)$

$$A = [\,\vec{a}_1 \quad \vec{a}_2 \quad \cdots \quad \vec{a}_n\,]$$

i.e., A can be partitioned into n column vectors

Ⓑ $A = B$

if and only if $\Leftrightarrow$ ① same size, and ② corresponding entries are =

Exs $\begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$

As vectors, maybe =.

$$\underset{(1\times 2)}{\begin{bmatrix} 1 & 0 \end{bmatrix}} \neq \underset{(2\times 1)}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Ⓒ $A + B$

To obtain $A+B$, add corresp. entries.

add

Ex $$\underbrace{\begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}}_{A} + \underbrace{\begin{bmatrix} -1 & -3 \\ 1 & 2 \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} 0 & -5 \\ 4 & 2 \end{bmatrix}}_{A+B}$$

(A, B must have the same size ("+" compatible), otherwise (o.w.))
If diff. sizes, $A+B$ is undefined.

$A - B$: subtract corresp. entries

(D) cA (Scalar Multiplication)

c ← a real # (scalar)

To obtain cA, multiply each entry of A by c.

Ex $\underset{c}{5}\underbrace{\begin{bmatrix} 2 & -\frac{3}{5} \\ 0 & 1 \end{bmatrix}}_{A} = \begin{bmatrix} 10 & -3 \\ 0 & 5 \end{bmatrix}$

Can do 1, 3

(E) (Row vector) times (Column vector)

$$[a_1 \ a_2 \ \cdots \ a_n] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = a_1b_1 + a_2b_2 + \ldots + a_nb_n$$

Add products of corresp. entries.
Think: Dot product

Ex $[1 \ 0 \ 3] \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$

Take the product of the 1st entries...

$= (1)(4) + (0)(5) + (3)(-1)$

$= 4 + 0 - 3$

$= 1$ or $[1]$

(scalar or maybe 1×1 matrix. [1])

Books sloppy if you treat them as matrices.

Can do 1, 3

(F) AB (Matrix Multiplication)

$$AB = \text{"}C\text{"} = [c_{ij}]$$

Seems weird componentwise mult. is diff. (.3 (E) - we'll see why.

where $c_{ij} = (i^{th}$ row of $A)$ times $(j^{th}$ column of $B)$

Ex

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\underset{2 \times 3}{A} \qquad \underset{3 \times 2}{B}$$

need = ("Mult. compatible"), o.w., AB is undefined

Trick

Some have seen

$$\begin{array}{cc} & \begin{bmatrix} 4 & -1 \\ 3 & 0 \\ 1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix} & \underbrace{\begin{matrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{matrix}}_{AB} \end{array}$$

$$c_{11} = (1)(4) + (-1)(3) + (0)(1) = 1$$

(You)

$$c_{12} = -1 + 0 + 0 = -1$$
$$c_{21} = 8 + 3 + 3 = 14$$
$$c_{22} = -2 + 0 + 6 = 4$$

$$AB = \begin{bmatrix} 1 & -1 \\ 14 & 4 \end{bmatrix}$$

$$\underbrace{\underset{m\times n}{A} \quad \underset{n\times p}{B}}_{m\times p}$$

$$\left(\begin{array}{cc} & \overset{1 \qquad p}{[\ B\ ]} \\ {}^{1}_{m}[\ A\ ] & [\ AB\ ] \end{array} \right)$$

what do you think the size of the product is

$n \times p$ and $m \times n$

Often, $AB \neq \underset{n\times p\ m\times n}{BA}$ (In fact, if $m \neq p$, BA is undefined.)

Matrix mult. is <u>not commutative</u>. ("+" is)

why are we doing it this way? A 1/3; Decomp A 1st all but 19

Read Ex on pp. 45-46 (Applic.)

$A^k = \underbrace{AA \cdots A}_{k \text{ copies}}$ (if A is square, k is a natural #)

6 Matrix Notation and Systems (of Linear Eqs)

Form $A\vec{x} = \vec{b}$ where the augmented matrix is $[A \,\vdots\, \vec{b}]$ $\begin{pmatrix}\text{coeff.}\\ \text{matrix}\end{pmatrix}$ (RHS)

Ex. 8 (1,2)

<u>Ex</u> $\begin{cases} 2x_1 + 4x_2 - 2x_3 = 0 \\ 3x_1 + 5x_2 \qquad = 1 \end{cases}$

Augmented matrix

$$\underbrace{\begin{bmatrix} 2 & 4 & -2 \\ 3 & 5 & 0 \end{bmatrix}}_{A} \Bigg| \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\vec{b}}$$

So, the system can be written as

$$\begin{bmatrix} 2 & 4 & -2 \\ 3 & 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A \qquad \vec{x} = \vec{b}$$

(Why?

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & -2 \\ 3 & 5 & 0 \end{bmatrix} \begin{bmatrix} 2x_1 + 4x_2 - 2x_3 \\ 3x_1 + 5x_2 \end{bmatrix} \begin{matrix} = 0 \\ = 1 \end{matrix}$$

)

Solve $[A \mid \vec{b}] \Rightarrow$

$$\begin{cases} x_1 = 2 - 5t \\ x_2 = -1 + 3t \\ x_3 = t \end{cases} \qquad t \text{ is any real \#}$$

seems awkward

$$\begin{cases} x_1 = 2 + t(-5) \\ x_2 = -1 + t(3) \\ x_3 = 0 + t(1) \end{cases}$$

Vector notation

$$\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}}_{\text{"Initial pt."}} + t \underbrace{\begin{bmatrix} -5 \\ 3 \\ 1 \end{bmatrix}}_{\text{"Direction vector"}}$$

Discuss later.

If I graph the sol'n set in 3-space, what do I get?

∞ many sol'ns

$$t = 0 \rightarrow \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$t = 1 \rightarrow \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

$$t = 2 \rightarrow \begin{bmatrix} -8 \\ 5 \\ 2 \end{bmatrix}$$

etc.