# 2.2: ALGEBRA OF MATRICES

Read (skip proof on p. 61)

## Ⓐ Matrix Eqs.

meaning that they're the same size

Ex Let $A, B, X$ be $m \times n$

Solve $B + 4X = 7A$ for $X$

Can "+" or "−" same matrix on both sides.

$$4X = 7A - B$$

"÷ by 4" awkward (for matrices)

Let's "mult. by $\frac{1}{4}$."

$$\frac{1}{4}(4X) = \frac{1}{4}(7A - B)$$

$$X = \frac{1}{4}(7A - B)$$ ← easier to compute

or (can distribute $\frac{1}{4}$)

$$X = \frac{7}{4}A - \frac{1}{4}B$$

## Ⓑ Matrix Mult(iplication) Properties

So that all additions, mults. are defined.

Assume size-compatibility.

① Associative

$$(AB)C = A(BC)$$

② Not commutative

$$AB \text{ may or may not } = BA$$

③ Distributive props.

$$A(B+C) = AB + AC$$
$$(A+B)C = AC + BC$$

Both true only possible if A, B, C - n×n

④ If $A = B$, then $AC = BC$ (if sizes compatible) ← mult. on the right by C

$CA = CB$ ← " left

⑤ But can't always cancel.

Algebra I: $ac = bc$

$\Rightarrow a = b$ (provided $c \neq 0$)

There are many sits. where $AC = BC$ but $A \neq B$.

If $AC = BC$,

$A$ may or may not $= B$

not just the matrix

Ex $\underset{A}{\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}} \underset{C}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} = \underset{AC}{\begin{bmatrix} 5 & 5 \\ 9 & 9 \end{bmatrix}}$

$\underset{B}{\begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}} \underset{C}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} = \underset{BC}{\begin{bmatrix} 5 & 5 \\ 9 & 9 \end{bmatrix}}$.

$AC = BC$, but $A \neq B$.

A counterexample to the claim $\underbrace{\text{"}AC = BC \rightarrow A = B\text{"}}_{\text{a "False" stmt, even though sometimes True.}}$

True means always true
False may mean sometimes false
(given the assumptions)

## Ⓒ Special Matrices

A zero matrix "$O$" has all entries $= 0$.

Ex $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ "$O_{2\times 3}$"

An identity matrix "$I$" is a square matrix with:

"1"s along the main diagonal

$$\begin{bmatrix} 1 & & O \\ & 1 & \\ O & & \ddots \; 1 \end{bmatrix}$$

"0"s elsewhere

$I_n$ = the $n \times n$ identity

Ex $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

If $A$ is $m \times n$,

$$\underset{(m\times n)}{A}\ \underset{(n\times n)}{I_n} = A$$

$$\underset{(m\times m)}{I_m}\ \underset{(m\times n)}{A} = A$$

Length of row = Length of col

$[4\ \ 5][\quad] = [4\ \ 5]$

<u>Ex</u>

$$[4\ \ 5]\underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{I_2} = [4\ \ 5]$$

<u>① $A^T$ (A Transpose)</u>

If $A$ is $m \times n$..

Interchange rows $\leftrightarrow$ columns to obtain $\underset{(n\times m)}{A^T}$.

<u>Ex</u> If $A = \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$ $(2\times 3)$

then $A^T = \begin{bmatrix}1 & 4\\2 & 5\\3 & 6\end{bmatrix}$ $(3\times 2)$

## Properties

(Assume sizes compatible)

① $(A^T)^T = A$ (Key)

It doesn't matter if you add 1st and then flip or..

② $(A+B)^T = A^T + B^T$

It doesn't matter if you 'c 1st or flip 1st.

③ $(cA)^T = c(A^T)$

④ $(AB)^T = B^T A^T$ (Key)

Transpose of a product = <u>Reverse</u> product of transposes

## (E) Symmetric Matrices

Square matrix "A" is <u>symmetric</u> $\leftrightarrow A = A^T$

<u>Ex</u> $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 3 & 5 \\ 4 & 5 & -1 \end{bmatrix}$ mirror

rows same as cols.

= what?

$a_{ij} = a_{ji}$ for all $1 \le i \le n$, $1 \le j \le n$