CH. 4: VECTOR SPACES

4.1: VECTORS IN $\mathbb{R}^n$

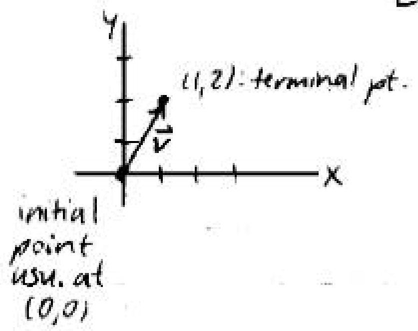
Ⓐ Vectors in the Plane ($\mathbb{R}^2$)

① Drawing

Book: (1, 2)

Ex $\vec{v} = (1, 2)$ or $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

In principle, you can move it around.



② Vector "+"

add corresp. components just like for general matrices

They weren't sure!

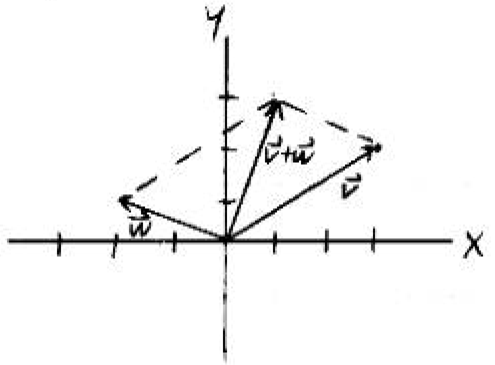
Ex If $\vec{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

then $\vec{v} + \vec{w} = \begin{bmatrix} 3 + (-2) \\ 2 + 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Geometrically

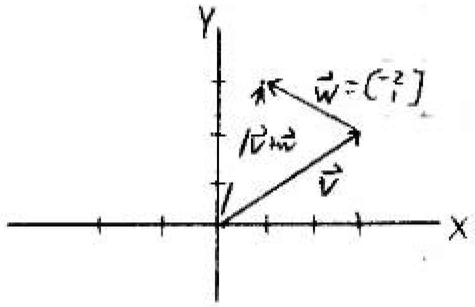
Parallelogram Law

$\vec{v}+\vec{w}$ represents the diagonal of the par. determined by $\vec{v}, \vec{w}$, provided initial pt. at (0,0) resultant



Triangle Law

We start the vector $\vec{w}$ here... Head-to-tail ? Tail-to-head ?

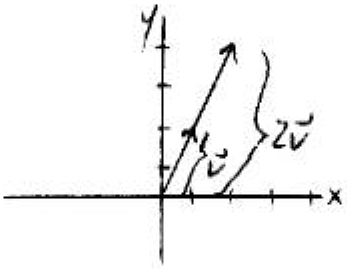


③ Scalar Mult.

Ex If $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

then $2\vec{v} = \begin{bmatrix} 2(1) \\ 2(2) \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

What do you think happens geometrically when you × a vector by 2? (Not ∥, specifically ↗ not ⤴)



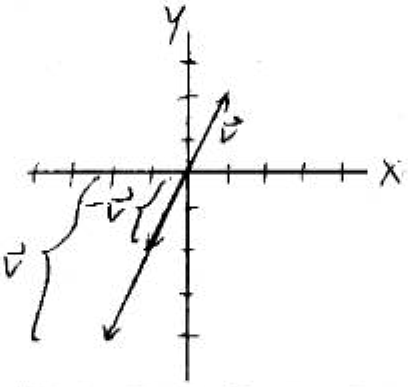
If $c > 0$,
$c\vec{v}$ has the same direction as $\vec{v}$ but is c times as long.

<u>Ex</u> If $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

the negative of $\vec{v}$

then $-\vec{v} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

and $-2\vec{v} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$



If $c < 0$,
$c\vec{v}$ and $\vec{v}$ have opposite directions.

surprisingly important

④ $\vec{0}$

$\vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in $\mathbb{R}^2$

0 multiplier zeros all the comps of $\vec{v}$

$0\vec{v} = \vec{0}$
$c\vec{0} = \vec{0}$

Key If $c\vec{v} = \vec{0}$, then $c = 0$ or $\vec{v} = \vec{0}$

(B) Vectors in $\mathbb{R}^n$

$\mathbb{R}^n$ = n-space

= set of all $\underbrace{\text{real ordered n-tuples}}_{(\text{"points" in } \mathbb{R}^n)}$

$$= \left\{ \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \,\middle|\, \text{"}x_i\text{"s are real \#s} \right\}$$

such that

Rules for $\mathbb{R}^2$ extend to $\mathbb{R}^3, \mathbb{R}^4$, etc.

<u>Ex</u> If $\vec{v} = \begin{bmatrix} 1 \\ 3 \\ -4 \\ 0 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 0 \\ 3 \\ 5 \\ -7 \end{bmatrix}$.

then $2\vec{v} - \vec{w} = ?$

$$2\vec{v} - \vec{w} = 2\begin{bmatrix} 1 \\ 3 \\ -4 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \\ 5 \\ -7 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -8 \\ 0 \end{bmatrix} + \begin{bmatrix} -0 \\ -3 \\ -5 \\ +7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 3 \\ -13 \\ 7 \end{bmatrix} \leftarrow \text{in } \mathbb{R}^4$$