Nonstandard Bases

<u>Review Ex</u> $T: V \to W$
$\quad\quad (\mathbb{R}^2) \quad (\mathbb{R}^2)$

$$T(x_1, x_2) = (x_1 - x_2,\ 3x_1) = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

B: Ordered basis for V

$$B = \left\{ \underset{\vec{v}_1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}, \underset{\vec{v}_2}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} \right\}$$

(standard <u>for now</u>)

B': " " W

Usually standard basis for W

$$B' = \left\{ \underset{\vec{w}_1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}, \underset{\vec{w}_2}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} \right\}$$

$$A = \Big[[T(\vec{v}_1)]_{B'} \ \ [T(\vec{v}_2)]_{B'} \Big] = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$$

If $\vec{v} = (5, 3)$, say $\Rightarrow$

$$A\vec{v} = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 15 \end{bmatrix}$$

applies T

✓: $T(\vec{v}) = (5-3,\ 3(5)) = (2, 15)$ or $\begin{bmatrix} 2 \\ 15 \end{bmatrix}$

Now

Say $B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$

$\qquad\quad \vec{v}_1 \quad \vec{v}_2$

Now, A = matrix for T relative to B and B'

$$= \Big[[T(\vec{v}_1)]_{B'} \quad [T(\vec{v}_2)]_{B'} \Big]$$

where:

if nonstandard, solve system

$$T(\vec{v}_1) = T(1,1) = (0,3) = \overbrace{0\vec{w}_1 + 3\vec{w}_2}$$

$$\Rightarrow [T(\vec{v}_1)]_{B'} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}_{B'}$$

$$T(\vec{v}_2) = T(2,1) = (1,6) = 1\vec{w}_1 + 6\vec{w}_2$$

$$\Rightarrow [T(\vec{v}_2)]_{B'} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}_{B'}$$

$$= \begin{bmatrix} 0 & 1 \\ 3 & 6 \end{bmatrix}$$

Then, $[T(\vec{v})]_{B'} = A [\vec{v}]_B$

applies T to "weird coordinates" of $\vec{v}$ relative to the "weird basis" B

result: the "ordinary coordinates" of $T(\vec{v})$ relative to the "ordinary basis" B'

$\underline{Ex}$ $B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$
$\quad\quad\quad\quad \vec{v}_1 \quad\quad \vec{v}_2$

$\vec{v} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ or $(5,3)$

Method 1 Observe: $\vec{v} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

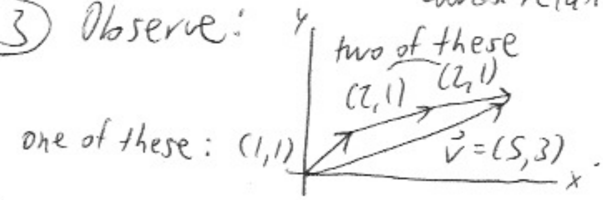
Method 2 Solve: $\vec{v} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

i.e., $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

coords. relative to "weird basis" B

Method 3 Observe:



In any case $\Rightarrow [\vec{v}]_B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\underset{\text{(standard)}}{[T(\vec{v})]_{B'}} = A[\vec{v}]_B = \begin{bmatrix} 0 & 1 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 15 \end{bmatrix}$, as in Review Ex

$\checkmark$: $T(5,3) = (2,15)$ or $\begin{bmatrix} 2 \\ 15 \end{bmatrix}$

$[\vec{v}]_{\text{standard}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \xrightarrow{T} \underset{\text{(standard)}}{[T(\vec{v})]_{B'}} = \begin{bmatrix} 2 \\ 15 \end{bmatrix}$