7.3: ORTHOGONALLY DIAGONALIZING SYMMETRIC MATRICES

(A) Symmetric Matrices

A - $n \times n$, real

A is symmetric $\Leftrightarrow$ $A^T = A$
(rows $\leftrightarrow$ cols)

Ex
$$\begin{bmatrix} 0 & 1 & \boxed{-2} \\ 1 & 3 & 5 \\ \boxed{-2} & 5 & -4 \end{bmatrix}$$

Book's term

Real Spectral Theorem

If A is (real) symmetric, then

(1) A is diag'e.
(An eval of (alg.) multiplicity k will have a k-dim espace. You can find n LI evecs.) } All alg. mults. = geom. mults. for all evals.

(2) All evals of A are real.
(The set of evals is called the spectrum of A.)

(B) Orthogonal Matrices

P - $n \times n$

: P is orthogonal (OG)

To find the inverse of an orthogonal matrix, you simply take its ^{T}!

We generally don't want to mess w/ P^{-1}.

$\overset{\text{Definition}}{\Longleftrightarrow}$ P is invertible and $P^{-1} = P^T$ - Nice property if you need to find P^{-1}!

$PP^{-1} = PP^T$

$I = PP^T$ (equivalent)

To show P is OG, Nice!

See Ex 5 (p. 407)

$\overset{\text{Theorem}}{\Longleftrightarrow}$ its column vectors form an orthonormal set, $n \times n$

(pairwise) orthogonal unit vectors

WARNING: An orthogonal matrix is a square matrix with orthonormal cols.

Ⓒ A is <u>orthagonally diagonalizable</u>

$\overset{\text{Def'n}}{\Longleftrightarrow}$ there exists an <u>orthagonal</u> matrix P such that $D = P^{-1}AP$

some diagonal matrix

if P is OG, how can we rewrite $P^{-1}AP$?

Reminder: to invert P, you just take its transpose

$(D = P^{T}AP)$

$\overset{\text{Theorem}}{\Longleftrightarrow}$ A is <u>symmetric</u>.

Ⓓ <u>How do you orthogonally diagonalize a (real) symmetric matrix A?</u>

Diagonalize A as usual, <u>except</u> you must <u>normalize</u> the $\vec{p}_i$'s.

what else must we do?

<u>Read Ex 8 (p. 410)</u>

$$A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

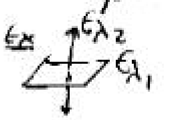
$$\lambda_1 = -3 \longrightarrow \vec{p}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \longrightarrow \vec{u}_1 = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

$$\lambda_2 = 2 \longrightarrow \vec{p}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \longrightarrow \vec{u}_2 = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

$P = [\vec{u}_1 \ \vec{u}_2]$

$= \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}$

Thm: If A is symmetric, evecs corresponding to different evals are $\perp$. Ex



$\vec{u}_1 \perp \vec{u}_2$

What if an espace has dim ≥ 2?

What can we use to get

Basis $\xrightarrow{\text{Gram-Schmidt}}$ Orthonormal basis

breathe

(Not in this class, at least for a Ch. 7 problem.)

If someone wanted to see e-thing it's easy to invert P - you just take the transpose

Let $D = P^{-1}AP$ or P^TAP } You don't have to work out! Just list the evals in a diagonal matrix like so

Do you have to work out P^TAP?

Then, $D = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$ λ_1 λ_2