Math 119 Name: _____

QUIZ 2

(LESSONS 11-18: DISCRETE PROBABILITY)
MATH 119 – FALL 2022 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate. You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.

THE FORMULA SHEET IS AT THE END.

1) (12 points). Two standard six-sided dice are rolled. One die is red; the other is green. The "number" on a die is the number of holes on the side that comes up. The "total" of the two dice is the sum of the numbers on the dice. Write your answers as **exact fractions**.

   • a) What is the probability of getting a 4 on the red die and a 4 on the green die? Your answer will be one fraction. (2 points)

   • b) What is the probability that the total is 6? Show work by using a grid as in class (you don't have to draw lines) or listing ordered pairs. (5 points)

• c) What is the probability of getting a 2 or a 5 on the red die or a 3 or a 4 on the green die? Your answer will be one fraction. Show work by using a grid as in class (you don't have to draw lines) or listing ordered pairs. (5 points)

2) (5 points). Two popular soda drinks are Poke and Cepsi. According to a study, 60% of American adults like Poke, 45% like Cepsi, and 30% like both. (Assume that the study is correct.) What is the probability that a randomly selected American adult likes Poke or Cepsi? Write your answer as a decimal, a percent, or a fraction.

3) (20 points). Professor Potter teaches two classes – a morning class and an evening class. All of Potter's 85 students take a test, and a student can either pass or fail the test. Consider the following two-way frequency (or contingency) table. Write your answers as **exact fractions**.

		Test Result		
		Pass	Fail	Total
Class	Morning	30	20	50
	Evening	25	10	35
	Total	55	30	85

• a) What is the **conditional probability** that a randomly selected student of Potter's is a morning student, **given that** the person passed the test?
(5 points)

• b) What is the **conditional probability** that a randomly selected student of Potter's passed the test, **given that** the person is a morning student?
(5 points)

• c) What is the probability that a randomly selected student of Potter's is in the evening class **or** passed the test? Your answer will be one fraction.
(5 points)

• d) What is the probability that a randomly selected student of Potter's is in the evening class **and** passed the test? Your answer will be one fraction.
(5 points)

4) (4 points). Four standard six-sided dice are rolled. What is the probability that all four dice come up "6"s? Your answer will be one fraction.

5) (8 points). Three cards are randomly drawn from a standard 52-card deck **without replacement** (each drawn card is immediately, permanently removed from the deck). What is the probability that all three cards are Queens? Write your answer as an exact fraction and also round it as a decimal to three significant figures. **Show work** by writing fractions and what you do with them! Hint: There are originally four Queens in the deck of cards.

6) (4 points). A student takes three classes. Let X = the number of classes the student will pass. The student gives the following **incomplete** probability distribution for X. Find the value of a to complete the probability distribution.

Value (x)	**Probability** $P(x)$
0	0.100
1	a
2	0.350
3	0.400

7) (12 points). You and your family first pay $500 for a life insurance policy for the year. If you die during the year, your family gets $20,000. If you do not die during the year, your family gets nothing (beyond your presence, at least). Let X = you and your family's net monetary gain as a result of the decision to purchase the policy. Based on research, you assume that your probability of dying during the year is 0.020. Assuming this is correct, find $E(X)$ and **interpret** it, as we did in class. Also, fill out the table:

Outcome for the year	**Value** (x)	**Probability** $P(x)$
You live.		
You die.		

8) (15 points). Showing some work or notation may help with partial credit. None of the answers is "one."

• a) You have nine candy bars in your Halloween bowl; all of them are different. A scary ghost comes to your door. You decide to give the ghost four of your candy bars. How many ways can you choose the four candy bars for the ghost? The order of the chosen candy bars does not matter. (5 points)

• b) Your neighbor also has nine candy bars in a Halloween bowl; all of them are different. Nine children form a line outside your neighbor's door. Your neighbor will give one candy bar to each child. How many ways can the nine candy bars be given to the nine children? (5 points)

• c) Billy Bonka makes nine different flavors of candy bars. You buy Billy Bonka bars at a large store. You decide to eat one Billy Bonka candy bar each day for the next five days. (Assume that every flavor is available to you on every day.) How many ways can you eat five Billy Bonka candy bars for the next five days if you only care about flavors and days? (5 points)

9) (5 points). A student answers all six questions on a multiple-choice math quiz. Each question has three possible options: "A," "B," or "C," only one of which is correct. The student guesses randomly on all questions. The random variable is the number of questions the student gets correct. As in class, give the distribution (including the type of distribution and the values of the two parameters) that best describes the random variable.

10) (15 points; 5 points each). It turns out that the random variable described in 9) above has the following probability distribution table.

Value (x)	**Probability** $P(x)$
0	0.088
1	0.263
2	0.329
3	0.219
4	0.082
5	0.016
6	0.001

Find the indicated probabilities regarding the number of questions the student gets correct. Write your answers to three decimal places. Showing work can help with partial credit; for example, rewriting using an inequality or rewriting as a sum or difference of probabilities.

- a) P(at most 2)

- b) P(at least 5)

- c) P(more than 3)

MATH 119: QUIZ 2 FORMULA SHEET

Probabilities for Events Involving Equally Likely Outcomes ("elos")

$$P(A) = \frac{\#\text{ of elos for which } A \text{ occurs}}{N}$$

Complementary Probabilities

$$P(\text{not } A) = P(\overline{A}) = P(A^C) = 1 - P(A)$$

Addition Rule for Mutually Exclusive Events ("mees")

$$P(A \text{ or } B) = P(A) + P(B)$$

General Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Multiplication Rule for Independent Events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$
$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$

General Multiplication Rule

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$
$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A \text{ and } B)$$

Conditional Probabilities

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(B \mid A) = \frac{\#(A \text{ and } B)}{\#(A)} = \frac{\#\text{ of trials (or "elos") in which } A \text{ and } B \text{ occur}}{\#\text{ of trials (or "elos") in which } A \text{ occurs}}$$

(SEE NEXT PAGE!)

Expected Value (or Mean) of a Probability Distribution

$$E(X), \text{ or } \mu = \sum P(x) \cdot x, \text{ or } \sum x \cdot P(x)$$

Variance and Standard Deviation of a Probability Distribution
• (NOT ON QUIZ 2)

$$VAR(X), \text{ or } \sigma^2 = \sum (x-\mu)^2 \cdot P(x), \text{ or } E(X^2) - \mu^2$$
$$SD(X), \text{ or } \sigma = \sqrt{VAR(X)}$$

Full (or Complete) Permutations of *n* Distinct Items

$$_nP_n = n!$$

(Other) Partial Permutations of *n* Distinct Items, Taken *r* at a Time
• (NOT ON QUIZ 2, aside from maybe *n*! itself)

$$_nP_r = \frac{n!}{(n-r)!}$$

Combinations of *n* Distinct Items, Taken *r* at a Time; Binomial Coefficients

$$_nC_r, \text{ or } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Binomial Probability Formula
• (NOT NEEDED ON QUIZ 2, BUT MAY HELP WITH CHECKING)

$$P(x) = {}_nC_x\, p^x q^{n-x}, \text{ or } \binom{n}{x} p^x q^{n-x}$$