Math 119 Name: _____

QUIZ 2

(LESSONS 11-18: DISCRETE PROBABILITY)
MATH 119 – SPRING 2022 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate. You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.

THE FORMULA SHEET IS AT THE END.

1) (12 points). Two standard six-sided dice are rolled. One die is red; the other is green. The "number" on a die is the number of holes on the side that comes up. Write your answers as **exact fractions**.

   • a) What is the probability of getting a 6 on the red die? (2 points)

   • b) What is the probability that the total is 9? The "total" of the two dice is the sum of the numbers on the dice. Show work by using a grid (you don't have to draw lines) or listing ordered pairs. (5 points)

   • c) What is the probability of getting doubles or a 4 on the red die? Your answer will be one fraction. You get "doubles" when the red die and the green die come up the same number. Show work by using a grid (you don't have to draw lines) or listing ordered pairs. (5 points)

2) (5 points). A school band is choosing the colors on a new uniform. 30% of the band members want orange to be on the uniform. 20% of the band members want purple to be on the uniform. 5% of the band members want both orange and purple to be on the uniform. What is the probability that a randomly selected band member wants orange or purple to be on the uniform? Write your answer as a decimal, a percent, or a fraction.

3) (20 points). Each adult in Voterville is a Democrat or a Republican (but not both). All 5000 adults in Voterville vote on a local proposition. Each adult votes "Yes" or "No" (but not both) and honestly tells the local newspaper how they voted. Consider the following two-way frequency (or contingency) table. Write your answers as **exact fractions**.

		Vote		
		Yes	No	Total
Party	Democrat	1500	500	2000
	Republican	1250	1750	3000
	Total	2750	2250	5000

• a) An adult in Voterville is randomly selected. What is the probability that the adult is a Democrat **and** the adult voted "Yes"? Your answer will be one fraction. (5 points)

• b) An adult in Voterville is randomly selected. What is the probability that the adult is a Democrat **or** the adult voted "Yes"? Your answer will be one fraction. (5 points)

• c) What is the **conditional probability** that a randomly selected adult in Voterville voted "Yes," **given that** the adult is a Republican? (5 points)

• d) What is the **conditional probability** that a randomly selected adult in Voterville is a Republican, **given that** the adult voted "Yes"? (5 points)

4) (15 points). A standard 52-card deck has 13 hearts, 13 diamonds, 13 clubs, and 13 spades.

• a) Four cards are randomly drawn from a standard 52-card deck **with replacement** (each drawn card is immediately returned to the deck). What is the probability that all four cards are spades? Write your answer as an exact fraction and also round it as a decimal to three significant figures. **Show work** by showing how you got your answer! (5 points)

• b) Four cards are randomly drawn from a standard 52-card deck **without replacement** (each drawn card is immediately, permanently removed from the deck). What is the probability that the first two cards are spades and the last two cards are clubs? Write your answer as an exact fraction and also round it as a decimal to three significant figures. **Show work** by writing fractions and what you do with them! (10 points)

5) (3 points). A mechanic is examining a car. The probability that none of the tires will need to be replaced in the next year is 0.9. What is then the probability that at least one of the tires will need to be replaced in the next year?

6) (10 points). The mechanic examines another car and assigns the following probability distribution for X, the number of tires on the car that will need to be replaced in the next year.

Value (x)	**Probability** $P(x)$
0	0.650
1	0.200
2	0.110
3	0.025
4	0.015

Find $E(X)$ and **interpret** it. Hint: Consider many cars in the same condition.

7) (15 points). Showing some work or notation may help with partial credit. None of the answers is "one." Each answer is a single number (of ways).

- a) You go to a grocery store and buy cereal, milk, and bread. The store has four brands of cereal, three brands of milk, and five brands of bread. If you only care about brands, how many ways can you buy cereal, milk, and bread at the store? (5 points)

- b) You buy ice cream at an ice cream store. The store has eight toppings and you will choose four of them for your ice cream. How many ways can you then choose the toppings? The order of the toppings does not matter. (5 points)

- c) You have seven masks, all with different colors, that you will use for a particular week. You use one mask per day during the week, and you do not reuse masks between days. If you only care about mask colors and which days you wear them on, how many ways are there for you to select your masks throughout the week? (5 points)

8) (5 points). According to a recent Pew Research Poll, 69% of U.S. adults favor admitting thousands of Ukrainian refugees into the U.S. (Assume that this poll is accurate.) Six U.S. adults are randomly selected for interviews. The random variable is the number of interviewed adults who favor admitting thousands of Ukrainian refugees into the U.S. As in class, give the distribution (including the type of distribution and the values of the two parameters) that best describes the random variable.

9) (15 points; 5 points each). It turns out that the random variable described in 8) above has the following probability distribution table, with probabilities rounded off to three decimal places.

Value (x)	**Probability** $P(x)$
0	0.000
1	0.012
2	0.066
3	0.196
4	0.327
5	0.291
6	0.108

We could write: $P(0) = 0+$, but don't worry about that.

Find the indicated probabilities regarding the number of interviewed adults who favor admitting thousands of Ukrainian refugees into the U.S. Showing work can help with partial credit; for example, rewriting using an inequality or rewriting as a sum or difference of probabilities.

- a) P(at most 3)

- b) P(more than 4)

- c) P(at least 3)

MATH 119: QUIZ 2 FORMULA SHEET

Probabilities for Events Involving Equally Likely Outcomes ("elos")

$$P(A) = \frac{\#\text{ of elos for which } A \text{ occurs}}{N}$$

Complementary Probabilities

$$P(\text{not } A) = P(\overline{A}) = P(A^C) = 1 - P(A)$$

Addition Rule for Mutually Exclusive Events ("mees")

$$P(A \text{ or } B) = P(A) + P(B)$$

General Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Multiplication Rule for Independent Events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$
$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$

General Multiplication Rule

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$
$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A \text{ and } B)$$

Conditional Probabilities

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(B \mid A) = \frac{\#(A \text{ and } B)}{\#(A)} = \frac{\#\text{ of trials (or "elos") in which } A \text{ and } B \text{ occur}}{\#\text{ of trials (or "elos") in which } A \text{ occurs}}$$

(SEE NEXT PAGE!)

Expected Value (or Mean) of a Probability Distribution

$$E(X), \text{ or } \mu = \sum P(x) \cdot x, \text{ or } \sum x \cdot P(x)$$

Variance and Standard Deviation of a Probability Distribution
• (NOT ON QUIZ 2)

$$VAR(X), \text{ or } \sigma^2 = \sum (x-\mu)^2 \cdot P(x), \text{ or } E(X^2) - \mu^2$$
$$SD(X), \text{ or } \sigma = \sqrt{VAR(X)}$$

Full (or Complete) Permutations of *n* Distinct Items

$$_nP_n = n!$$

(Other) Partial Permutations of *n* Distinct Items, Taken *r* at a Time
• (NOT ON QUIZ 2, aside from maybe *n*! itself)

$$_nP_r = \frac{n!}{(n-r)!}$$

Combinations of *n* Distinct Items, Taken *r* at a Time; Binomial Coefficients

$$_nC_r, \text{ or } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Binomial Probability Formula
• (NOT NEEDED ON QUIZ 2, BUT MAY HELP WITH CHECKING)

$$P(x) = {}_nC_x\, p^x q^{n-x}, \text{ or } \binom{n}{x} p^x q^{n-x}$$