QUIZ 3
(LESSONS 19-23: CONTINUOUS PROBABILITY)
MATH 119 – FALL 2019 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate. You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.

THE FORMULA SHEET IS AT THE END.

1) (2 points). If \( X \sim \text{Uniform}\left[0, 1\right] \), find \( P(0.5 < X < 0.9) \).

2) (4 points). If \( X \sim \text{Uniform}\left[500, 1000\right] \), find \( P(600 < X < 800) \).

Write \( z \) scores out to two decimal places. Write probabilities to four decimal places.

3) (17 points). Software tells you that \( P(Z < -1.50) \approx 0.0668 \).
   
   - a) Sketch a figure clearly showing this fact, as in class. Shade in the relevant region. (3 points)

   - b) Find \( P(Z > -1.50) \). Explain why (show work) and sketch a figure clearly showing this, as in class. Shade in the relevant region. (7 points)

   - c) Find \( P(Z > 1.50) \). Explain why and sketch a figure clearly showing this, as in class. Shade in the relevant region. (7 points)
4) (46 points). This information is for parts a) through e).

A large lecture class takes a test. The scores are approximately normally distributed with mean 70 points and standard deviation 8 points. Let \( X \) be the score of a randomly selected student in the class.

(Note: The professor gives partial credit so that scores such as 70.1 points, 70.2 points, etc. are possible.)

Use these hints regarding the \( Z \) distribution:

- a) Find \( P(X < 60 \text{ points}) \). First write the corresponding probability expression for \( Z \). Show work by using the Formula for \( z \) Scores. (6 points)

- b) Find \( P(X > 60 \text{ points}) \). First write the corresponding probability expression for \( Z \). (6 points)
• c) Find \( P(60 \text{ points} < X < 76 \text{ points}) \). First write the corresponding probability expression for \( Z \). Use the Formula for \( z \) Scores when showing work. You may use your work from part a). (8 points)

• d) Find the 25\(^{th}\) percentile (also known as the 1\(^{st}\) quartile) of the distribution of test scores; round it off to the nearest tenth of a point (one decimal place). Write the corresponding probability expression for \( X \) and interpret it. Hint: The 25\(^{th}\) percentile of the \( Z \) distribution is about \(-0.67\). (8 points)
• e) Four students who took the test are randomly selected. We will find the probability that the average of their scores \( \overline{X} \) was less than 60 points. That is, we will find \( P(\overline{X} < 60 \text{ points}) \). We will compare our answer to part a). (18 points)

Do these steps:

First write the approximate sampling distribution for \( \overline{X} \).

Write the probability expression for \( Z \) corresponding to \( P(\overline{X} < 60 \text{ points}) \). Show work by using the Formula for \( z \) Scores for Sample Means.

Find \( P(\overline{X} < 60 \text{ points}) \).

Compare the result to the one from part a), where we cared about the score of just one random student; is your result here higher or lower than the one in part a)?
5) A student answers all the questions on a multiple-choice test with 100 questions. Each question has four possible options: “A,” “B,” “C,” or “D,” only one of which is correct. The student guesses randomly on all questions. The random variable $X$ is the number of questions the student gets correct. Approximate the probability that the student gets at least 20 questions correct, $P(X \geq 20)$. Apply an appropriate continuity correction. Follow these steps: (31 points)

- a) **Describe the distribution of** $X$, as in class. (5 points)

- b) Verify that a **normal approximation** to the distribution of $X$ would be appropriate, as in class. (4 points)

- c) **Describe the normal distribution** that can be used to approximate the distribution of $X$, as in class. Round off to five significant figures if necessary. (8 points)
• d) Apply a continuity correction and rewrite \( P(X \geq 20) \) in terms of \( X_c \), as in class. (4 points)

• e) Find the z score for the boundary value of \( x_c \) using the Formula for z Scores. (4 points)

• f) Write the probability expression for \( Z \) corresponding to your expression for \( X_c \) from part d), as in class. (2 points)

• g) Approximate \( P(X \geq 20) \), as in class. (4 points)
Use these hints regarding the Z distribution:
Probabilities for a Continuous Uniform[0, 1] Distribution

Let $X \sim \text{Uniform}[0, 1]$. If $0 \leq x_1 \leq x_2 \leq 1$, then $P(x_1 < X < x_2) = x_2 - x_1$.

Probabilities for a Continuous Uniform[a, b] Distribution

Let $X \sim \text{Uniform}[a, b]$. If $a \leq x_1 \leq x_2 \leq b$, then $P(x_1 < X < x_2) = \frac{x_2 - x_1}{b - a}$.

Probability Density Functions for Normal and Standard Normal Distributions
• (NOT NEEDED ON QUIZ 3)

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Formula for z Scores

$$z = \frac{x - \text{mean}}{\text{SD}} = \frac{x - \mu}{\sigma}$$ (We usually round off z to two decimal places.)

Formula for Transforming z Scores Into x Scores

$$x = \mu + z \sigma$$

Standard Error (SE) of the Sample Mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

“Two SE” $(2\sigma_{\bar{x}})$ Rule for Usual Values of a Sample Mean

An appropriate interval of usual values for a sample mean is given by $\left(\mu - 2\sigma_{\bar{x}}, \mu + 2\sigma_{\bar{x}}\right)$.

Formula for z Scores for Sample Means

$$z = \frac{\bar{x} - \text{mean}}{\text{SE}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}, \quad \text{or} \quad \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$ (Round off z to two decimal places.)

(SEE NEXT PAGE!)
Central Limit Theorem (CLT) for Means (Averages); Sampling Distribution for $\bar{X}$

Assume $X \sim D$ with mean $\mu$ and finite SD $\sigma$.

Let $\bar{X}$ be the sample mean of $n$ iid (independent and identically distributed) results from $D$.

If the sample size $n$ is large ($n > 30$), or if $D$ is approximately normal, then

$$\bar{X} \sim \text{N} \left( \text{mean} = \mu_{\bar{X}} = \mu, \ SD \ or \ SE = \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \right)$$

Central Limit Theorem (CLT) for Sums (NOT NEEDED ON QUIZ 3)

This is similar, but let $\sum X$ be the sample sum of $n$ iid results from $D$.

If the sample size $n$ is large ($n > 30$), or if $D$ is approximately normal, then

$$\sum X \sim \text{N} \left( \text{mean} = \mu_{\sum X} = n\mu, \ SD = \sigma_{\sum X} = \sigma\sqrt{n} \right)$$

Conditions for Using Normal Approximations to Binomial Distributions

We will use normal approximations to binomial distributions if and only if the following conditions apply:

- $np \geq 5$, and
- $nq \geq 5$

Mean and SD of a Binomial Distribution

If $X \sim \text{Bin} \left( n, p \right)$, then:

- mean, $\mu = np$
- SD, $\sigma = \sqrt{npq}$

Using Normal Approximations to Binomial Distributions

Let $X \sim \text{Bin} \left( n, p \right)$. If $np \geq 5$ and $nq \geq 5$, then:

$$X \sim \text{N} \left( \text{mean} = \mu = np, \ SD = \sigma = \sqrt{npq} \right)$$

Continuity Corrections for Normal Approximations to Binomial Distributions

We associate the integer value $a$ in the binomial distribution with the interval $(a - 0.5, a + 0.5)$ in the approximating normal distribution. Think: “rounding.”