Math 119 Name: ________________________

QUIZ 3

(LESSONS 19-23: CONTINUOUS PROBABILITY)
MATH 119 – SPRING 2022 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate. You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.

THE FORMULA SHEET IS AT THE END; FEEL FREE TO TEAR IT OFF.

1) (2 points). If $X \sim \text{Uniform}\left[0,1\right]$, find $P(0.4 < X < 0.7)$.
You may write your answer as a decimal, percent, or fraction.

2) (4 points). If $X \sim \text{Uniform}[60, 90]$, find $P(70 < X < 75)$.
You may write your answer as a fraction or as a decimal or percent rounded to three significant figures.

Write z scores out to two decimal places. Write probabilities to four decimal places.

3) (17 points). Software tells you that $P(Z < 1.25) \approx 0.8944$.

• a) Sketch a figure clearly showing this fact, as in class. Shade in the relevant region. (3 points)

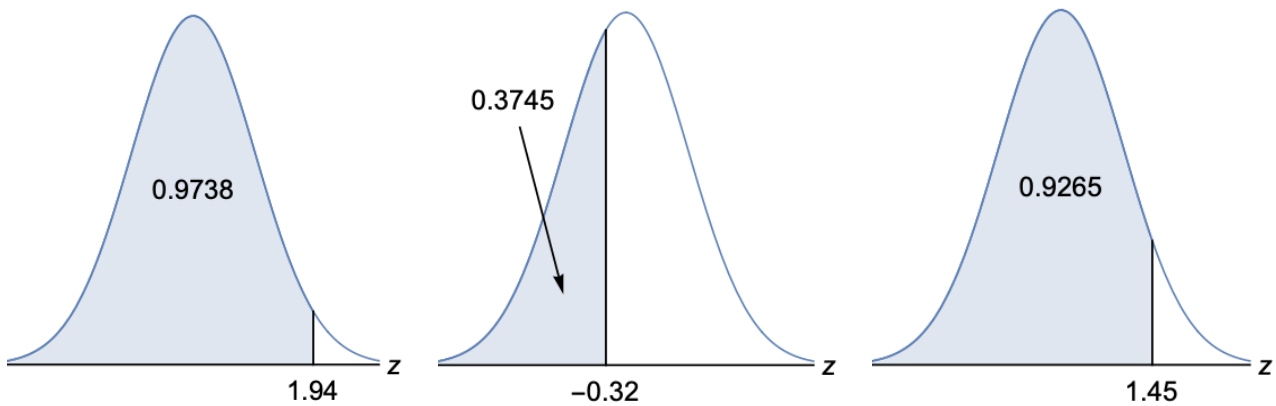
• b) Find $P(Z > 1.25)$. Explain why (show work) and sketch a figure clearly showing this, as in class. Shade in the relevant region. (7 points)

• c) Find $P(Z > -1.25)$. Explain why and sketch a figure clearly showing this, as in class. Shade in the relevant region. (7 points)

4) (46 points). This information is for parts a) through e).

Assume that body temperatures of healthy humans are approximately normally distributed with mean 98.20 degrees Fahrenheit (^{o}F) and standard deviation 0.62 degrees Fahrenheit (^{o}F). Let X be the body temperature of a randomly selected healthy human.

Use these hints regarding the Z distribution:



- a) The medical community says that someone has a fever if they have a body temperature higher than 100.40 degrees Fahrenheit (^{o}F). If a healthy human has a body temperature of 100.40 degrees Fahrenheit (^{o}F), find the corresponding z score rounded off to two decimal places. Show work by using the Formula for z Scores.

  Note: The z score would be considered unusual for a healthy human. (3 points)

- b) The CEO of a company is very worried about COVID. The CEO says that any human whose body temperature is higher than 99.40 degrees Fahrenheit (^{o}F) cannot enter their building. Find $P(X > 99.40\,^{o}\text{F})$, which is the probability that a randomly selected healthy human would not be allowed to enter the building for this reason. First write the corresponding **probability expression for Z**. Use the Formula for z Scores when showing work. (9 points)

• c) Find $P(98.00\,^{o}\text{F} < X < 99.40\,^{o}\text{F})$. First write the corresponding **probability expression for $\boldsymbol{Z}$**. Use the Formula for *z* Scores when showing work. You may use your work from part b). (8 points)

• d) The CEO of the company later decides to use the 99$^{\text{th}}$ percentile of the distribution of healthy human body temperatures as the cutoff for not being able to enter the building. Find this 99$^{\text{th}}$ percentile and round it off to two decimal places in degrees Fahrenheit. Write the corresponding **probability statement for $\boldsymbol{X}$** and **interpret** it. Hint: The 99$^{\text{th}}$ percentile of the standard normal *Z* distribution is about 2.33. (8 points)

• e) Nine healthy humans are randomly selected. We will find the probability that the average of their body temperatures $\left(\overline{X}\right)$ is higher than 98.50 degrees Fahrenheit (°F). That is, we will find $P\left(\overline{X} > 98.50\ ^{\circ}\text{F}\right)$. (18 points)

Do these steps:

First write the approximate **sampling distribution** for $\overline{X}$.
When rounding off, round off to five significant figures.

Write the **probability expression for *Z*** corresponding to $P\left(\overline{X} > 98.50\ ^{\circ}\text{F}\right)$
Show work by using the Formula for *z* Scores for Sample Means.

Find $P\left(\overline{X} > 98.50\ ^{\circ}\text{F}\right)$.

Check: It turns out that, for **one** random healthy human, the probability that their body temperature is higher than 98.50 degrees Fahrenheit (°F) is about 0.3142. That is, $P\left(X > 98.50\ ^{\circ}\text{F}\right) \approx 0.3142$. The probability that the **average of nine** healthy human body temperatures is higher than 98.50 degrees Fahrenheit (°F) should be **lower**, since we're talking about the probability mass in a tail that excludes the mean. That is, your answer to $P\left(\overline{X} > 98.50\ ^{\circ}\text{F}\right)$ should be **lower than 0.3142**.

5) A student answers all the questions on a multiple-choice test with 60 questions. Each question has five possible options: “A,” “B,” “C,” “D,” or “E,” only one of which is correct. The student guesses randomly on all questions. The random variable X is the number of questions the student gets correct. Approximate the probability that the student gets more than 15 questions correct, $P(X > 15)$. Apply an appropriate continuity correction. Follow these steps: (31 points)

- a) **Describe the distribution of *X***, as in class. (5 points)

- b) **Verify** that a **normal approximation** to the distribution of X would be appropriate, as in class. (4 points)

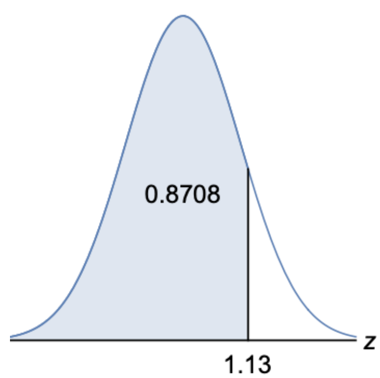
- c) **Describe the normal distribution** that can be used to approximate the distribution of X, as in class. Round off to five significant figures if necessary. (8 points)

• d) **Apply a continuity correction** and rewrite $P(X > 15)$ in terms of X_c, as in class. (4 points)

• e) Find the z **score for the boundary value of** x_c; show work by using the Formula for z Scores and round it off to two decimal places. (4 points)

• f) Write the **probability expression for** Z corresponding to your expression for X_c from part d), as in class. (2 points)

• g) Approximate $P(X > 15)$, as in class. (4 points)
Use these hints regarding the Z distribution:



MATH 119: QUIZ 3 FORMULA SHEET

Probabilities for a Continuous Uniform[0, 1] Distribution

Let $X \sim \text{Uniform}[0,1]$. If $0 \le x_1 \le x_2 \le 1$, then $P(x_1 < X < x_2) = x_2 - x_1$.

Probabilities for a Continuous Uniform[*a*, *b*] Distribution

Let $X \sim \text{Uniform}[a,b]$. If $a \le x_1 \le x_2 \le b$, then $P(x_1 < X < x_2) = \dfrac{x_2 - x_1}{b-a}$.

Probability Density Functions for Normal and Standard Normal Distributions
• (NOT NEEDED ON QUIZ 3)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Formula for *z* Scores

$$z = \frac{x - \text{mean}}{\text{SD}} = \frac{x-\mu}{\sigma}$$ (We usually round off *z* to two decimal places.)

Formula for Transforming *z* Scores Into *x* Scores

$$x = \mu + z\sigma$$

Standard Error (SE) of the Sample Mean

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

"Two SE" $\left(2\sigma_{\overline{X}}\right)$ Rule for Usual Values of a Sample Mean

An appropriate interval of usual values for a sample mean is given by $\left(\mu - 2\sigma_{\overline{X}},\ \mu + 2\sigma_{\overline{X}}\right)$.

Formula for *z* Scores for Sample Means

$$z = \frac{\overline{x} - \text{mean}}{\text{SE}} = \frac{\overline{x} - \mu_{\overline{X}}}{\sigma_{\overline{X}}}, \text{ or } \frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$ (Round off *z* to two decimal places.)

(SEE NEXT PAGE!)

Central Limit Theorem (CLT) for Means (Averages); Sampling Distribution for $\overline{X}$

Assume $X \sim D$ with mean μ and finite SD σ.

Let $\overline{X}$ be the sample mean of *n* <u>iid</u> (independent and identically distributed) results from *D*.

If the sample size *n* is large $(n > 30)$, or if *D* is approximately normal, then

$$\overline{X} \overset{\text{approx.}}{\sim} \mathrm{N}\left(\text{mean} = \mu_{\overline{X}} = \mu, \ \text{SD or SE} = \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} \right)$$

Central Limit Theorem (CLT) for Sums (NOT NEEDED ON QUIZ 3)

This is similar, but let $\sum X$ be the sample sum of *n* <u>iid</u> results from *D*.

If the sample size *n* is large $(n > 30)$, or if *D* is approximately normal, then

$$\sum X \overset{\text{approx.}}{\sim} \mathrm{N}\left(\text{mean} = \mu_{\sum X} = n\mu, \ \text{SD} = \sigma_{\sum X} = \sigma\sqrt{n} \right)$$

Conditions for Using Normal Approximations to Binomial Distributions

We will use normal approximations to binomial distributions if and only if the following conditions apply:

- $np \geq 5$, and
- $nq \geq 5$

Mean and SD of a Binomial Distribution

If $X \sim \mathrm{Bin}(n, p)$, then:

mean, $\mu = np$

SD, $\sigma = \sqrt{npq}$

Using Normal Approximations to Binomial Distributions

Let $X \sim \mathrm{Bin}(n, p)$. If $np \geq 5$ and $nq \geq 5$, then:

$$X \overset{\text{approx.}}{\sim} \mathrm{N}\left(\text{mean} = \mu = np, \ \ \text{SD} = \sigma = \sqrt{npq} \right)$$

Continuity Corrections for Normal Approximations to Binomial Distributions

We associate the integer value *a* in the **binomial** distribution with the interval $(a - 0.5, \ a + 0.5)$ in the approximating **normal** distribution. Think: "**rounding**."