Math 119 Name: ________________________

QUIZ 3

(LESSONS 19-23: CONTINUOUS PROBABILITY)
MATH 119 – SPRING 2023 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.
You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.
Do not leave decimal parts in fractions or fractions in fractions, such as .1/.2 or (1/3)/(2/3).

THE FORMULA SHEET IS AT THE END; FEEL FREE TO TEAR IT OFF.

1) (2 points). If $X \sim \text{Uniform}\left[0,1\right]$, find $P\left(0.5 < X < 0.9\right)$.
You may write your answer as a decimal, percent, or fraction.

2) (4 points). If $X \sim \text{Uniform}\left[500,1000\right]$, find $P\left(600 < X < 800\right)$.
You may write your answer as a decimal, percent, or fraction.

Write *z* scores out to two decimal places. Write probabilities to four decimal places.

3) (17 points). Software tells you that $P\left(Z < -1.50\right) \approx 0.0668$.

- a) Sketch a figure clearly showing this fact, as in class. Shade in the relevant region. (3 points)

- b) Find $P\left(Z > -1.50\right)$. Explain why (show work) and sketch a figure clearly showing this, as in class. Shade in the relevant region. (7 points)

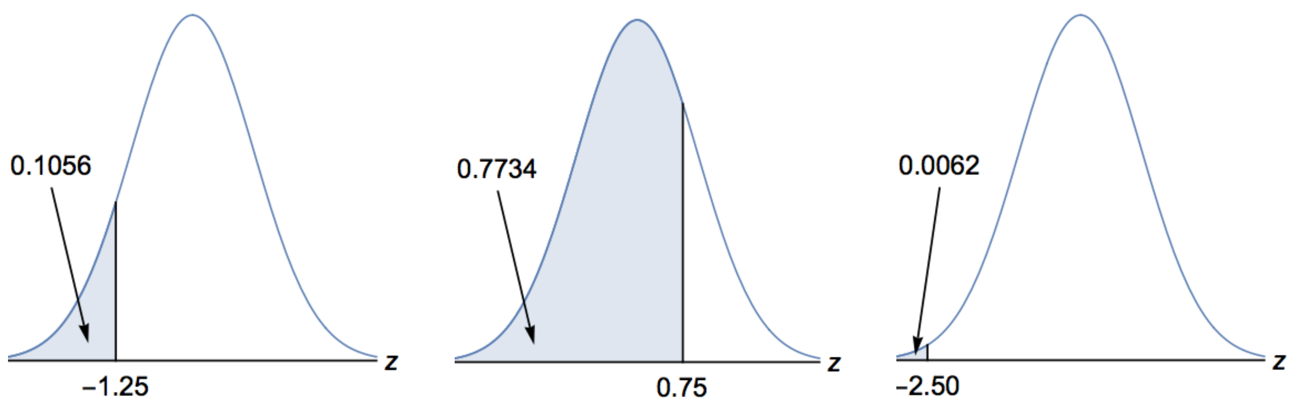
- c) Find $P\left(Z > 1.50\right)$. Explain why and sketch a figure clearly showing this, as in class. Shade in the relevant region. (7 points)

4) (46 points). This information is for parts a) through e).

A large lecture class takes a test. The scores are approximately normally distributed with mean 70 points and standard deviation 8 points. Let X be the score of a randomly selected student in the class.

(Note: The professor gives partial credit so that scores such as 70.1 points, 70.2 points, etc. are possible.)

Use these hints regarding the Z distribution:



• a) Find $P(X < 60 \text{ points})$. First write the corresponding **probability expression for Z**. Show work by using the Formula for z Scores. (6 points)

• b) Find $P(X > 60 \text{ points})$. First write the corresponding **probability expression for Z**. (6 points)

• c) Find $P(60 \text{ points} < X < 76 \text{ points})$. First write the corresponding **probability expression for Z**. Use the Formula for z Scores when showing work. You may use your work from part a). (8 points)

• d) Find the 25th percentile (also known as the 1st quartile) of the distribution of test scores; round it off to the nearest tenth of a point (one decimal place). Write the corresponding **probability expression for X** and **interpret** it. Hint: The 25th percentile of the Z distribution is about -0.67. (8 points)

• e) Four students who took the test are randomly selected. We will find the probability that the average of their scores $\left(\overline{X}\right)$ was less than 60 points. That is, we will find $P\left(\overline{X} < 60 \text{ points}\right)$. We will compare our answer to part a). (18 points)

Do these steps:

First write the approximate **sampling distribution** for $\overline{X}$.

Write the **probability expression for *Z*** corresponding to $P\left(\overline{X} < 60 \text{ points}\right)$
Show work by using the Formula for *z* Scores for Sample Means.

Find $P\left(\overline{X} < 60 \text{ points}\right)$.

Compare the result to the one from part a), where we cared about the score of just one random student; is your result here higher or lower than the one in part a)?

5) A student answers all the questions on a multiple-choice test with 100 questions. Each question has four possible options: “A,” “B,” “C,” or “D,” only one of which is correct. The student guesses randomly on all questions. The random variable X is the number of questions the student gets correct. Approximate the probability that the student gets at least 20 questions correct, $P\left(X \geq 20\right)$. Apply an appropriate continuity correction. Follow these steps: (31 points)

- a) **Describe the distribution of *X***, as in class. (5 points)

- b) **Verify** that a **normal approximation** to the distribution of X would be appropriate, as in class. (4 points)

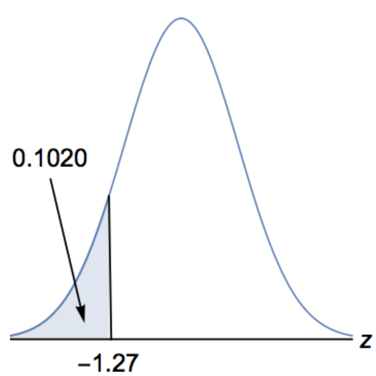
- c) **Describe the normal distribution** that can be used to approximate the distribution of X, as in class. Round off to five significant figures if necessary. (8 points)

• d) **Apply a continuity correction** and rewrite $P(X \geq 20)$ in terms of X_c, as in class. (4 points)

• e) Find the ***z* score for the boundary value of** $\boldsymbol{x_c}$; show work by using the Formula for *z* Scores and round it off to two decimal places. (4 points)

• f) Write the **probability expression for** $\boldsymbol{Z}$ corresponding to your expression for X_c from part d), as in class. (2 points)

• g) Approximate $P(X \geq 20)$, as in class. (4 points)
Use these hints regarding the Z distribution:



MATH 119: QUIZ 3 FORMULA SHEET

Probabilities for a Continuous Uniform[0, 1] Distribution

Let $X \sim \text{Uniform}[0,1]$. If $0 \le x_1 \le x_2 \le 1$, then $P(x_1 < X < x_2) = x_2 - x_1$.

Probabilities for a Continuous Uniform[*a*, *b*] Distribution

Let $X \sim \text{Uniform}[a,b]$. If $a \le x_1 \le x_2 \le b$, then $P(x_1 < X < x_2) = \dfrac{x_2 - x_1}{b-a}$.

Probability Density Functions for Normal and Standard Normal Distributions
• (NOT NEEDED ON QUIZ 3)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad\qquad f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Formula for *z* Scores

$$z = \frac{x - \text{mean}}{\text{SD}} = \frac{x-\mu}{\sigma}$$ (We usually round off *z* to two decimal places.)

Formula for Transforming *z* Scores Into *x* Scores

$$x = \mu + z\sigma$$

Standard Error (SE) of the Sample Mean

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

"Two SE" $\left(2\sigma_{\overline{X}}\right)$ Rule for Usual Values of a Sample Mean

An appropriate interval of usual values for a sample mean is given by $\left(\mu - 2\sigma_{\overline{X}},\ \mu + 2\sigma_{\overline{X}}\right)$.

Formula for *z* Scores for Sample Means

$$z = \frac{\overline{x} - \text{mean}}{\text{SE}} = \frac{\overline{x} - \mu_{\overline{X}}}{\sigma_{\overline{X}}}, \text{ or } \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$ (Round off *z* to two decimal places.)

(SEE NEXT PAGE!)

Central Limit Theorem (CLT) for Means (Averages); Sampling Distribution for $\overline{X}$

Assume $X \sim D$ with mean μ and finite SD σ.

Let $\overline{X}$ be the sample mean of *n* iid (independent and identically distributed) results from *D*.

If the sample size *n* is large $(n > 30)$, or if *D* is approximately normal, then

$$\overline{X} \overset{\text{approx.}}{\sim} \text{N}\left(\text{mean} \ = \ \mu_{\overline{X}} \ = \ \mu, \ \ \text{SD or SE} \ = \ \sigma_{\overline{X}} \ = \ \frac{\sigma}{\sqrt{n}} \right)$$

Central Limit Theorem (CLT) for Sums (NOT NEEDED ON QUIZ 3)

This is similar, but let $\sum X$ be the sample sum of *n* iid results from *D*.
If the sample size *n* is large $(n > 30)$, or if *D* is approximately normal, then

$$\sum X \overset{\text{approx.}}{\sim} \text{N}\left(\text{mean} \ = \ \mu_{\sum X} \ = \ n\mu, \ \ \text{SD} \ = \ \sigma_{\sum X} \ = \ \sigma\sqrt{n} \right)$$

Conditions for Using Normal Approximations to Binomial Distributions

We will use normal approximations to binomial distributions
if and only if the following conditions apply:

- $np \geq 5$, and
- $nq \geq 5$

Mean and SD of a Binomial Distribution

If $X \sim \text{Bin}(n, p)$, then:

mean, $\mu = np$

SD, $\sigma = \sqrt{npq}$

Using Normal Approximations to Binomial Distributions

Let $X \sim \text{Bin}(n, p)$. If $np \geq 5$ and $nq \geq 5$, then:

$$X \overset{\text{approx.}}{\sim} \text{N}\left(\text{mean} = \mu = np, \ \ \text{SD} = \sigma = \sqrt{npq} \right)$$

Continuity Corrections for Normal Approximations to Binomial Distributions

We associate the integer value *a* in the **binomial** distribution with the interval $(a - 0.5, \ a + 0.5)$ in the approximating **normal** distribution. Think: "**rounding**."