Math 119 Name: ________________________

QUIZ 4

(LESSONS 24-32: ESTIMATING POPULATION PARAMETERS;
INTRO TO HYPOTHESIS TESTING)
MATH 119 – SPRING 2022 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.
You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.

THE FORMULA SHEETS ARE AT THE END; FEEL FREE TO TEAR OFF.

1) (12 points). A news reporter wants to know the population mean of annual salaries for the employees of a large company in 2021. The reporter takes a random sample of 100 annual employee salaries at the company in 2021. The sample mean $\bar{x}$ is \$38,500. For a 95% confidence interval (CI) for μ, the population mean of annual salaries for the company's employees in 2021, the margin of error E is found to be \$15,000.

- a) What is a **point estimate** for the **population mean** of annual salaries for the company's employees in 2021? (2 points)

- b) What is the **lower limit** of the 95% CI for μ? (2 points)

- c) What is the **upper limit** of the 95% CI for μ? (2 points)

- d) **Write the 95% CI** for μ in terms of the values of $\bar{x}$ and E. (2 points)

- e) **Interpret** the **95% CI** for μ, as in class. (2 points)

- f) Would a **90% CI** be wider or smaller than the 95% CI for μ? (2 points)

2) (8 points). An insurance company uses software to predict the life expectancies of the residents in a large city. The company wants to know μ, the population mean life expectancy of the city's residents (based on the software). 50 of the city's residents are randomly selected, and the software predicts their life expectancies. We obtain $(72.4 \text{ years}, 78.0 \text{ years})$ as a 90% confidence interval (CI) for μ.

- a) What is the **sample mean** $\bar{x}$? (2 points)

- b) What is the **margin of error** E for the CI? (2 points)

- c) **Write the CI** in terms of the values of $\bar{x}$ and E. (2 points)

- d) **Interpret** the CI, as in class. (2 points)

3) (2 points). What is α for a 99% CI?

4) (8 points). Consider any of the t distributions.

- a) What is the **mean**? (2 points)

- b) Is the **standard deviation** equal to 1, less than 1, or greater than 1? (2 points)

- c) Yes or No: Is the distribution **symmetric** about its mean? (2 points)

- d) As the number of degrees of freedom (df) increases, what distribution will the t distributions approach? (2 points)

5) (4 points). Consider any of the χ^2 distributions.

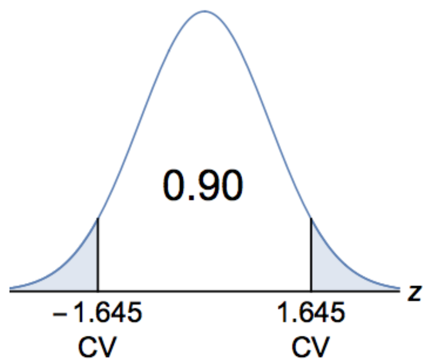
- a) Yes or No: Is the **mean** equal to 0? (2 points)

- b) Yes or No: Is the distribution **symmetric** about its mean? (2 points)

6) (2 points). In most basic applications, how many degrees of freedom (df) do we use for t and χ^2 distributions if the sample size is 40?

7) (7 points). The insurance company from Problem 2) uses new software to predict the life expectancies of the residents in the city. We would like to know μ, the population mean life expectancy of the city's residents (based on the new software). Assume that the population standard deviation (SD) is 11.8 years, which was the sample standard deviation from the study in Problem 2) involving the old software. **Find the required sample size** n that would give us a margin of error of 2.0 years for a 90% confidence interval (CI) for the population mean. Clearly show how this is obtained by plugging into an appropriate formula.

Use these hints about the z distribution:



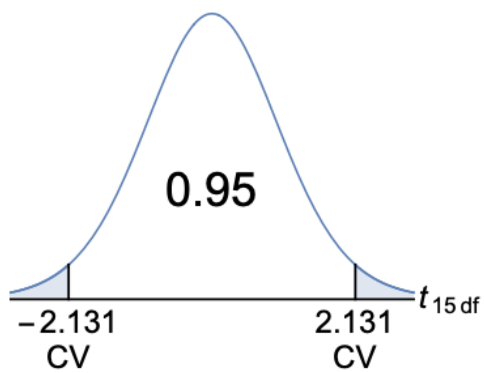
8) (2 points). The population standard deviation (SD) of life expectancies for the United States is more like 15 years. If we use this as a **conservative** estimate for the population standard deviation (SD) for the city, we would get a required sample size of at least 153 people if we wanted a 90% confidence interval (CI) for the population mean life expectancy for the city and a margin of error of 2.0 years. Your answer to Problem 7) should have given us the minimum required sample size if the population standard deviation (SD) for the city were only 11.8 years. Box in the correct answer:

• 153 people is **more than** the correct minimum required sample size from Problem 7).

• 153 people is **fewer than** the correct minimum required sample size from Problem 7).

9) (14 points). The governor of a state claims that the gas stations in the state have an average regular gas price of \$5.00 per gallon. A random sample of 16 gas stations gives us a sample mean of \$5.15 per gallon and a sample standard deviation (SD) of \$0.20 per gallon. Assume that regular gas prices for gas stations in the state are approximately normally distributed. You will find a 95% confidence interval (CI) for the population mean of regular gas prices for gas stations in the state.

Use these hints about the t distribution on 15 degrees of freedom (df):



- a) Why do we use a t distribution instead of a z distribution in this problem? Box in one: (2 points)

  § We are estimating a population mean, the population standard deviation is assumed to be known, and the sample size is large enough.

  § We are estimating a population mean, the population standard deviation is assumed to be unknown, and we assume that the regular gas prices for the state's gas stations are approximately normally distributed.

- b) **Write the 95% CI** in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula. (5 points)

- c) **Write the 95% CI** in the form $\left(\text{lower limit, upper limit}\right)$. (3 points)

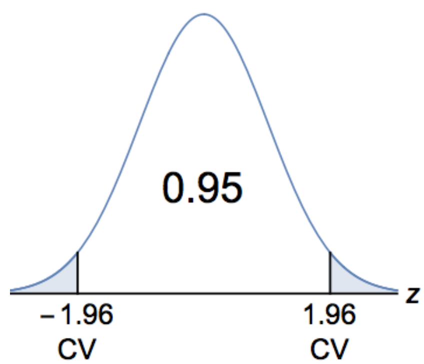
• d) **Interpret** the CI, as in class. (2 points)

• e) H_0 states that the average regular gas price for gas stations in the state is \$5.00 per gallon. This is the governor's claim. Use the significance level: $\alpha = 0.05$. Based on the 95% CI (and assuming that we are doing a two-tailed hypothesis test, as in the homework), do we decide to **reject** or **not reject** H_0? (2 points)

10) (28 points). A magician's coin is flipped 400 times. It comes up heads 230 times. You will find a 95% confidence interval (CI) for *p*, the probability that the coin comes up heads on a flip.

Round off values of $\hat{p}$, $\hat{q}$, and *E* to three decimal places.

Use these hints about the *z* distribution:



• a) Find the sample proportion of heads, $\hat{p}$. (3 points)

• b) Find the sample proportion of tails, $\hat{q}$. (3 points)

• c) **Verify** that normal approximations are appropriate in this problem. (4 points)

• d) **Write the 95% CI** in the form $p = \hat{p} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula. (7 points)

• e) **Write the 95% CI** in the form $\left(\text{lower limit, upper limit}\right)$. (3 points)

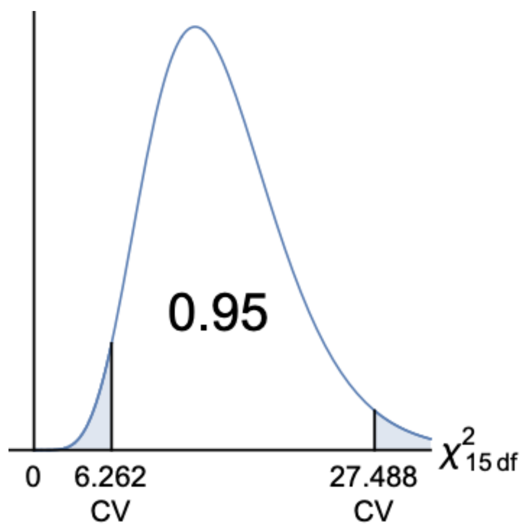
• f) **Interpret** the CI. (2 points)

• g) We want to know the probability that the coin comes up heads on a flip. **Find the required sample size** n that would give us a margin of error of 0.02 for a 95% confidence interval (CI) for this probability. Clearly show how this is obtained by plugging into an appropriate formula (the one giving us a conservative estimate). (6 points)

11) (10 points). Energy analysts in the state from Problem 9) also wanted to know the population standard deviation of the regular gas prices for the gas stations in the state. Assume that regular gas prices for gas stations in the state are approximately normally distributed. A random sample of 16 gas stations gives us a sample standard deviation (SD) of $0.20 per gallon.

- a) **Find** a 95% confidence interval (CI) for the population standard deviation (SD) of the regular gas prices for the gas stations in the state. Clearly show how the limits are obtained by plugging into an appropriate formula. **Write** the CI in either the form $\text{lower limit} < \sigma < \text{upper limit}$ or $(\text{lower limit, upper limit})$. (8 points)

  Use these hints about the χ^2 distribution on 15 degrees of freedom (df):



- b) **Interpret** the CI, as in class. (2 points)

12) (3 points). A magician's coin will be flipped 100 times. H_0 states that the coin is **fair**. H_1 states that the coin is **not fair**. Let's say we observe 60 heads among the 100 flips. Let X = the number of heads in 100 flips of a fair coin. Use the significance level: $\alpha = 0.05$.

Using a one-tailed *P*-value analysis,

$$\text{One-tailed } P\text{-value} = P(X \geq 60)$$
$$\approx 0.0284, \text{or } 2.84\%$$

Based on this one-tailed *P*-value, do we decide to **reject** or **not reject** H_0?

MATH 119: QUIZ 4 FORMULA SHEETS

Sample Proportion of Successes

$$\hat{p} = \frac{x}{n}$$

$(1-\alpha)$ **Confidence Interval (CI) for μ, where σ is Known**

(Assume the Central Limit Theorem (CLT) applies.)

$$\mu = \bar{x} \pm E$$

where the **margin of error**

$$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

That is,

$$\mu = \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Rounding Rule for E

Round off (or up) the margin of error E to the **same number of decimal places** as the given value of $\bar{x}$.

Determining Sample Size n for Estimating μ

For a $(1-\alpha)$ **confidence level** and a desired **margin of error** E,
the required sample size n is given by:

$$n = \left\lceil \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2 \right\rceil$$

where $\lceil \ \rceil$ is the ceiling (or "round-up") operator.

(SEE NEXT PAGE!)

$(1-\alpha)$ Confidence Interval (CI) for μ, where σ is Unknown

(Assume the Central Limit Theorem (CLT) applies.)

$$\mu = \bar{x} \pm E$$

where the **margin of error**

$$E = t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

That is,

$$\mu = \bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

We use the t distribution on $(n-1)$ **degrees of freedom (df)**.

<u>Rounding Rule for E</u>

Round off (or up) the margin of error E to the **same number of decimal places** as the given value of $\bar{x}$.

$(1-\alpha)$ Confidence Interval (CI) for p

(Assume $X \sim \text{Bin}(n, p)$. To justify a **normal approximation**, verify: $n\hat{p} \geq 5$, and $n\hat{q} \geq 5$.)

$$p = \hat{p} \pm E$$

where the **margin of error**

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

That is,

$$p = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

(SEE NEXT PAGE!)

Determining Sample Size n for Estimating p

For a $\left(1-\alpha\right)$ **confidence level** and a desired **margin of error** E,
the **conservative** estimate for the required sample size n is given by:

$$n = \left\lceil \frac{\left(z_{\alpha/2}\right)^2 \left(0.25\right)}{E^2} \right\rceil$$

where $\lceil \quad \rceil$ is the ceiling (or "round-up") operator.

$\left(1-\alpha\right)$ Confidence Intervals (CIs) for σ^2 and σ

(Assume $X \overset{\text{approx.}}{\sim}$ Normal.)

$$\frac{\left(n-1\right)s^2}{\chi_R^2} < \sigma^2 < \frac{\left(n-1\right)s^2}{\chi_L^2}$$

$$\sqrt{\frac{\left(n-1\right)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{\left(n-1\right)s^2}{\chi_L^2}}$$

We use the χ^2 distribution on $\left(n-1\right)$ **degrees of freedom (df)**.

χ_R^2 is the right (greater) CV.

χ_L^2 is the left (lesser) CV.

Rounding Rule for the Limits of the CIs

Round off (or "out") the limits of the CIs to the **same number of decimal places** as the given value of s^2 or s.