

QUIZ 4

(LESSONS 24-31: ESTIMATING POPULATION PARAMETERS)
MATH 119 – SPRING 2024 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate. You do not have to reduce fractions. For example, $10/20$ does not have to be rewritten as $\frac{1}{2}$.

THE FORMULA SHEETS ARE AT THE END; FEEL FREE TO TEAR OFF.

- 1) (12 points). We want to know the average number of units taken by students at a college this term. We randomly sample 70 students at the college this term. The sample mean \bar{x} is 10.8 units. For a 90% confidence interval (CI) for μ , the population mean number of units taken by students at the college this term, the margin of error E is found to be 0.6 units.
- a) What is a **point estimate** for the **population mean** number of units taken by students at the college this term? (2 points)

 - b) What is the **lower limit** of the 90% CI for μ ? (2 points)

 - c) What is the **upper limit** of the 90% CI for μ ? (2 points)

 - d) **Write the 90% CI** for μ in terms of the values of \bar{x} and E . (2 points)

 - e) **Interpret the 90% CI** for μ , as in class. (2 points)

 - f) Based on the same sample, would a **99% CI** be wider or smaller than the 90% CI for μ ? (2 points)

2) (8 points). A random sample of 45 quarters in circulation in the U.S. are weighed. A bank wants to know μ , the population mean weight (in grams) of all quarters in circulation in the U.S. We obtain (5.604 grams, 5.636 grams) as a 95% confidence interval (CI) for μ .

- a) What is the **sample mean** \bar{x} ? (2 points)

- b) What is the **margin of error** E for the 95% CI for μ ? (2 points)

- c) **Write the 95% CI for μ** in terms of the values of \bar{x} and E . (2 points)

- d) **Interpret the 95% CI for μ** , as in class. (2 points)

3) (2 points). What is α for an 80% CI?

4) (8 points). Consider any of the t distributions.

- a) What is the **mean**? (2 points)

- b) Is the **standard deviation** equal to 1, less than 1, or greater than 1? (2 points)

- c) Yes or No: Is the distribution **symmetric** about its mean? (2 points)

- d) As the number of degrees of freedom (df) increases, what distribution will the t distributions approach? (2 points)

5) (8 points). Consider the χ^2 distributions.

- a) Yes or No: Consider any of the χ^2 distributions.

Is the **mean** equal to 0? (2 points)

- b) Yes or No: Consider any of the χ^2 distributions.

Is the distribution **symmetric** about its mean? (2 points)

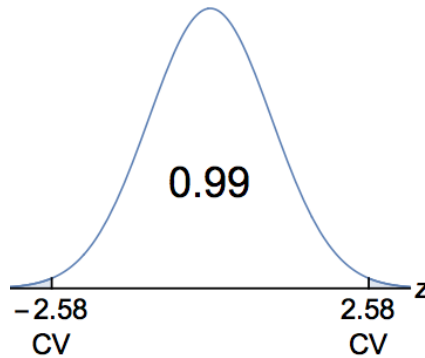
- c) Yes or No: As the number of degrees of freedom gets larger and approaches infinity, do the χ^2 distributions approach **a normal bell shape**? (2 points)

- d) Yes or No: As the number of degrees of freedom gets larger and approaches infinity, do the χ^2 distributions approach **the standard normal distribution**? (2 points)

6) (2 points). In most basic applications, how many degrees of freedom (df) do we use for t and χ^2 distributions if the sample size is 90?

7) (7 points). The college in Problem 1) defines a “working student” as a student who works a total of at least 20 hours per week at one or more jobs. The college would like to know μ , the population mean number of units taken by its working students this term. **Find the required sample size n** that would give us a margin of error of 0.5 units for a 99% confidence interval (CI) for the population mean. Assume that the population standard deviation (SD) is 3.1 units, which was the sample standard deviation from the study in Problem 1); this will likely give us a conservative estimate. Clearly show how this is obtained by plugging into an appropriate formula.

Use these hints about the z distribution:

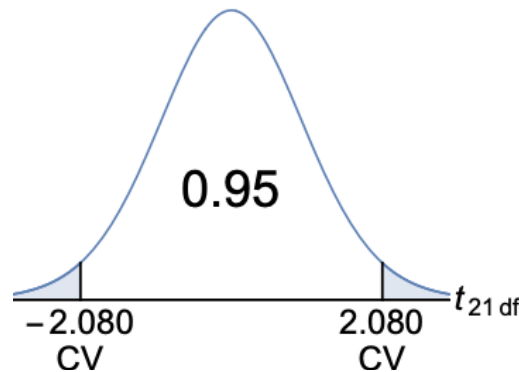


8) (2 points). In Problem 7), we want a higher confidence level and a lower margin of error compared to Problem 1), but we are basically assuming the same variation as in Problem 1). Box in the correct answer:

- The correct minimum required sample size from Problem 7) is **more than 70** students, the sample size from Problem 1).
- The correct minimum required sample size from Problem 7) is **fewer than 70** students, the sample size from Problem 1).

9) (14 points). A pharmaceutical company claims that its blood pressure pill lowers systolic blood pressure in American adults. A random sample of 22 American adults participate in a study on the pill. The drops in systolic blood pressure in the sample have mean 5.32 mm Hg and standard deviation (SD) 3.11 mm Hg. Assume that drops in systolic blood pressure among American adults who use the pill are approximately normally distributed. You will find a 95% confidence interval (CI) for μ , the population mean drop in systolic blood pressure among American adults taking the pill.

Use these hints about the t distribution on 21 degrees of freedom (df):



• a) We are estimating a population mean, and we assume that drops in systolic blood pressure among American adults who use the pill are approximately normally distributed (so the Central Limit Theorem applies here). Why do we use a t distribution instead of a z distribution in this problem? Box in one: (2 points)

§ The population standard deviation is assumed to be unknown.

§ The population standard deviation is assumed to be known.

• b) **Write the 95% CI** for μ in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula and round E to two decimal places. (7 points)

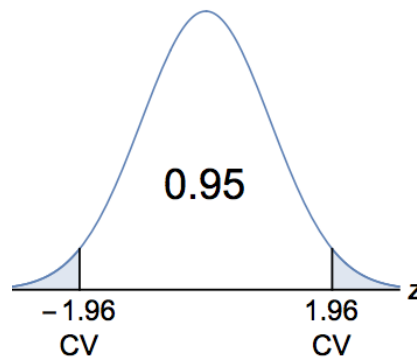
• c) Write the 95% CI for μ in the form (lower limit, upper limit). (3 points)

• d) Interpret the CI from b) and c), as in class. (2 points)

10) (27 points). A magician's coin is flipped 700 times. It comes up heads 383 times. You will find a 95% confidence interval (CI) for p , the probability that the coin comes up heads on a flip.

Round off values of \hat{p} , \hat{q} , and E to three decimal places.

Use these hints about the z distribution:



• a) Find the sample proportion of heads, \hat{p} . (2 points)

• b) Find the sample proportion of tails, \hat{q} . (2 points)

• c) Verify that normal approximations are appropriate in this problem, as in class. (4 points)

• d) **Write the 95% CI** for p in the form $p = \hat{p} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula. (8 points)

• e) **Write the 95% CI** for p in the form (lower limit, upper limit). (3 points)

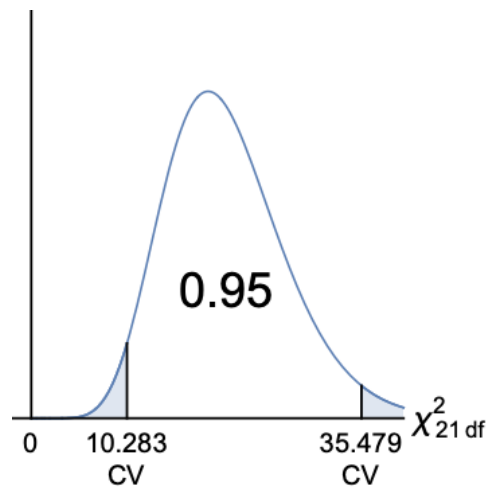
• f) **Interpret** the CI from d) and e), as in class. (2 points)

• g) We want to know the probability that the coin comes up heads on a flip. **Find the required sample size n** that would give us a margin of error of 0.01 for a 95% confidence interval (CI) for this probability. Clearly show how this is obtained by plugging into an appropriate formula (the one giving us a conservative estimate). (6 points)

11) (10 points). The pharmaceutical company from Problem 9) also wanted to know the population standard deviation of the drops in systolic blood pressure among American adults who use its pill. Assume that such drops are approximately normally distributed. A random sample of 22 American adults who take the pill has a standard deviation (SD) of 3.11 mm Hg.

- a) **Find** a 95% confidence interval (CI) for σ , the population standard deviation (SD) of the drops in systolic blood pressure among American adults who use the pill. Clearly show how the limits are obtained by plugging into an appropriate formula. **Write** the CI in either the form lower limit $< \sigma <$ upper limit or (lower limit, upper limit). Round the limits to two decimal places. (8 points)

Use these hints about the χ^2 distribution on 21 degrees of freedom (df):



- b) **Interpret** the CI, as in class. (2 points)

MATH 119: QUIZ 4 FORMULA SHEETS

Sample Proportion of Successes

$$\hat{p} = \frac{x}{n}$$

$(1-\alpha)$ Confidence Interval (CI) for μ , where σ is Known

(Assume the Central Limit Theorem (CLT) applies.)

$$\mu = \bar{x} \pm E$$

where the **margin of error**

$$E = z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right)$$

That is,

$$\mu = \bar{x} \pm z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right)$$

Rounding Rule for E

Round off (or up) the margin of error E to the **same number of decimal places** as the given value of \bar{x} .

Determining Sample Size n for Estimating μ

For a $(1-\alpha)$ **confidence level** and a desired **margin of error** E , the required sample size n is given by:

$$n = \left\lceil \left(\frac{z_{\alpha/2} \sigma}{E} \right)^2 \right\rceil$$

where $\lceil \quad \rceil$ is the ceiling (or “round-up”) operator.

(SEE NEXT PAGE!)

$(1-\alpha)$ Confidence Interval (CI) for μ , where σ is Unknown

(Assume the Central Limit Theorem (CLT) applies.)

$$\mu = \bar{x} \pm E$$

where the **margin of error**

$$E = t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right)$$

That is,

$$\mu = \bar{x} \pm t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right)$$

We use the t distribution on $(n-1)$ **degrees of freedom (df)**.

Rounding Rule for E

Round off (or up) the margin of error E to the **same number of decimal places** as the given value of \bar{x} .

$(1-\alpha)$ Confidence Interval (CI) for p

(Assume $X \sim \text{Bin}(n, p)$. To justify a **normal approximation**, verify: $n\hat{p} \geq 5$, and $n\hat{q} \geq 5$.)

$$p = \hat{p} \pm E$$

where the **margin of error**

$$E = z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

That is,

$$p = \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

(SEE NEXT PAGE!)

Determining Sample Size n for Estimating p

For a $(1 - \alpha)$ **confidence level** and a desired **margin of error** E , the **conservative** estimate for the required sample size n is given by:

$$n = \left\lceil \frac{(z_{\alpha/2})^2 (0.25)}{E^2} \right\rceil$$

where $\lceil \quad \rceil$ is the ceiling (or “round-up”) operator.

$(1 - \alpha)$ Confidence Intervals (CIs) for σ^2 and σ

(Assume $X \overset{\text{approx.}}{\sim}$ Normal.)

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$
$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

We use the χ^2 distribution on $(n - 1)$ **degrees of freedom (df)**.

χ_R^2 is the right (greater) CV.

χ_L^2 is the left (lesser) CV.

Rounding Rule for the Limits of the CIs

Round off (or “out”) the limits of the CIs to the **same number of decimal places** as the given value of s^2 or s .