Stat C1000 Name: ________________________

QUIZ 4

(LESSONS 24-31: ESTIMATING POPULATION PARAMETERS)
MATH 119 – SPRING 2026 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.
You do <u>not</u> have to reduce fractions. For example, 10/20 does <u>not</u> have to be rewritten as ½.

When rounding intermediate results, round to at least five significant digits.

<u>THE FORMULA SHEETS ARE AT THE END; FEEL FREE TO TEAR OFF.</u>

1) (12 points). Second Republic Bank is a local bank with many customers, none of whom are so wealthy that they really skew the figures. Let's say that the "account balance" a customer has with the bank includes both their checking and savings accounts. A reporter wants an interval estimate for μ, the **population mean** of account balances for the bank's customers. A random **sample** of the bank's customers is interviewed and asked for their account balances; assume that they are telling the truth. The sample mean $\bar{x}$ is \$5500. The margin of error E for a 95% confidence interval (CI) for μ is \$1500.

• a) What is a **point estimate** for the **population mean** of the account balances of the bank's customers? (2 points)

• b) What is the **lower limit** of the 95% CI for μ? (2 points)

• c) What is the **upper limit** of the 95% CI for μ? (2 points)

• d) **Write the 95% CI** for μ in terms of the values of $\bar{x}$ and E. (2 points)

• e) **Interpret** the CI from d), as in class. (2 points)

• f) Based on the same sample, would a **99% CI** be wider or smaller than the 95% CI for μ? (2 points)

2) (8 points). Third Republic Bank is another bank in another town. Let μ be the average account balance of this bank's customers. Based on a random sample of this bank's customers, we obtain ($2500, $6500) as a 90% confidence interval (CI) for μ.

- a) What is the **sample mean** $\bar{x}$? (2 points)

- b) What is the **margin of error** *E* for the 90% CI for μ? (2 points)

- c) **Write the 90% CI** for μ in terms of the values of $\bar{x}$ and *E*. (2 points)

- d) **Interpret** the CI from c), as in class. (2 points)

3) (2 points). What is α for a 99% confidence interval (CI)?

4) (8 points). Consider any of the *t* distributions.

- a) What is the **mean**? (2 points)

- b) Is the **standard deviation** equal to 1, less than 1, or greater than 1? (2 points)

- c) Yes or No: Is the distribution **symmetric** about its mean? (2 points)

- d) As the number of degrees of freedom (df) increases, what distribution will the *t* distributions approach? (2 points)

5) (8 points). Consider the χ^2 distributions.

- a) Yes or No: Consider any of the χ^2 distributions.
Is the **mean** equal to 0? (2 points)

- b) Yes or No: Consider any of the χ^2 distributions.
Is the distribution **symmetric** about its mean? (2 points)

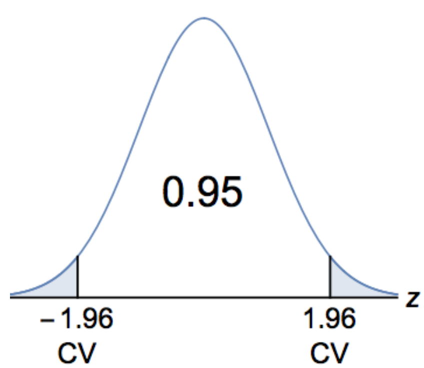
- c) Yes or No: As the number of degrees of freedom gets larger and approaches infinity, do the χ^2 distributions approach **a normal bell shape**? (2 points)

- d) Yes or No: As the number of degrees of freedom gets larger and approaches infinity, do the χ^2 distributions approach **the standard normal distribution**? (2 points)

6) (2 points). In most basic applications, how many degrees of freedom (df) do we use for t and χ^2 distributions if the sample size is 50?

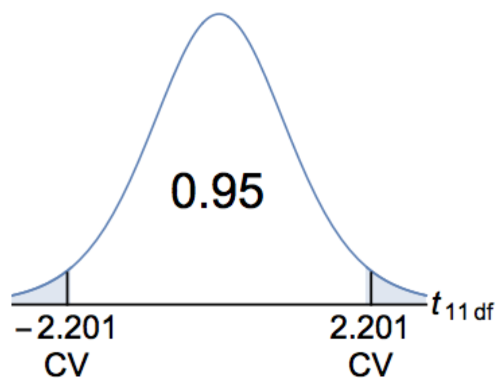
7) (7 points). We would like to know the population mean weight of adult Fredonian men. Assume that the population standard deviation (SD) is 30 pounds, which was the sample standard deviation from a study conducted ten years ago. **Find the required sample size** n that would give us a margin of error of 5 pounds for a 95% confidence interval (CI) for the population mean. Clearly show how this is obtained by plugging into an appropriate formula.

Use these hints about the z distribution:



8) (15 points). Banana Inc. describes "normal usage" of its cell phone, and it claims that the battery on the cell phone will last 10.00 hours on average under normal usage. A random sample of 12 Banana cell phone users who engage in normal usage participate in a survey and have their 12 phones monitored. Assume that Banana phone battery lifetimes under normal usage are approximately normally distributed. The sample mean battery lifetime is 8.52 hours, and the sample standard deviation (SD) is 0.68 hours. You will find a 95% confidence interval (CI) for μ, the population mean battery lifetime for the Banana cell phone under normal usage.

Use these hints about the t distribution on 11 degrees of freedom (df):



- a) We are estimating a population mean. Why do we use a t distribution instead of the z distribution in this problem? Box in one: (2 points)

  § The population standard deviation is assumed to be unknown, and we assume that the battery lifetimes are approximately normally distributed.

  § The sample size is large, the sample mean is known, and we are estimating a population standard deviation.

- b) **Write the 95% CI** for μ in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula, and round E to two decimal places. (7 points)

- c) **Write the 95% CI** for μ in the form $(\text{lower limit, upper limit})$. (3 points)

• d) **Interpret** the CI from b) and c), as in class. (2 points)

• e) Does the 95% CI for μ contain 10.00 hours, the mean claimed by Banana Inc. for the lifetime of its cell phone battery under normal usage? Note: This is related to a "two-tailed hypothesis test" with significance level: $\alpha = 0.05$. Box in one: (1 point)

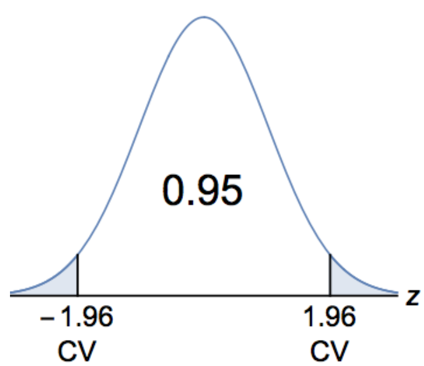
§ Yes, the 95% CI for μ **contains** 10.00 hours. Then, we would not reject Banana Inc.'s claim in the hypothesis test.

§ No, the 95% CI for μ **does not contain** 10.00 hours. Then, we would reject Banana Inc.'s claim in the hypothesis test.

9) (28 points). A magician's coin is flipped 300 times. It comes up heads 195 times. You will find a 95% confidence interval (CI) for *p*, the probability that the coin comes up heads on a flip.

Round off values of $\hat{p}$, $\hat{q}$, and E to three decimal places.

Use these hints about the *z* distribution:



• a) Find the sample proportion of heads, $\hat{p}$. (3 points)

• b) Find the sample proportion of tails, $\hat{q}$. (3 points)

• c) **Verify** that normal approximations are appropriate in this problem, as in class. (4 points)

• d) **Write the 95% CI** for p in the form $p = \hat{p} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula. Round E as a decimal to three decimal places. (7 points)

• e) **Write the 95% CI** for p in the form $(\text{lower limit, upper limit})$. (3 points)

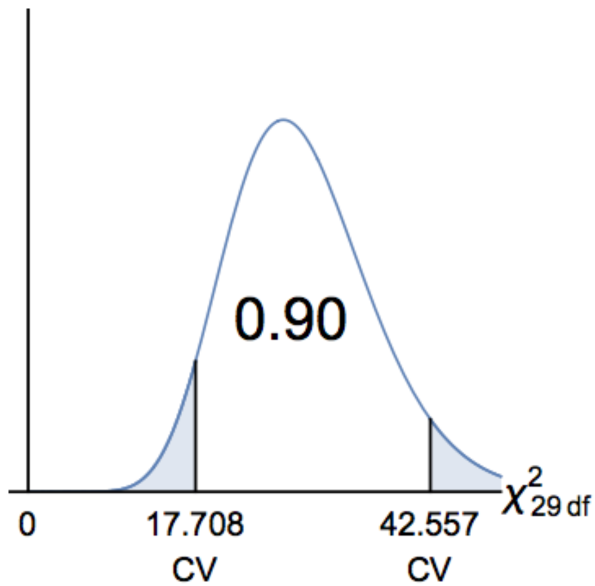
• f) **Interpret** the CI from d) and e), as in class. (2 points)

• g) We want to know the probability that the coin comes up heads on a flip. **Find the required sample size** *n* that would give us a margin of error of 0.03 for a 95% confidence interval (CI) for this probability. Clearly show how this is obtained by plugging into an appropriate formula (the one giving us a conservative estimate). (6 points)

10) (10 points). We would like to know the population standard deviation of the weights of adult Fredonian men. Assume that the weights of adult Fredonian men are approximately **normally distributed**. We randomly select 30 adult Fredonian men. The sample variance is 1005 square pounds.

- a) **Find** a 90% confidence interval (CI) for σ, the population standard deviation (SD) of the weights of adult Fredonian men. Clearly show how the limits are obtained by plugging into an appropriate formula. **Write** the CI in either the form $\text{lower limit} < \sigma < \text{upper limit}$ or $(\text{lower limit, upper limit})$. Round the limits to integers (that is, to zero decimal places). (8 points)

Use these hints about the χ^2 distribution on 29 degrees of freedom (df):



- b) **Interpret** the CI from a), as in class. (2 points)

STAT C1000: QUIZ 4 FORMULA SHEET

Sample Proportion of Successes

$$\hat{p} = \frac{x}{n}$$

$(1-\alpha)$ Confidence Interval (CI) for μ, where σ is Known

(Assume the Central Limit Theorem (CLT) applies.)

$$\mu = \bar{x} \pm E$$

where the **margin of error**

$$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

That is,

$$\mu = \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Rounding Rule for E

Round off (or up) the margin of error E to the **same number of decimal places** as the given value of $\bar{x}$.

Determining Sample Size n for Estimating μ

For a $(1-\alpha)$ **confidence level** and a desired **margin of error** E, the required sample size n is given by:

$$n = \left\lceil \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 \right\rceil$$

where $\lceil \quad \rceil$ is the ceiling (or "round-up") operator.

(SEE NEXT PAGE!)

$(1-\alpha)$ Confidence Interval (CI) for μ, where σ is Unknown

(Assume the Central Limit Theorem (CLT) applies.)

$$\mu = \bar{x} \pm E$$

where the **margin of error**

$$E = t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

That is,

$$\mu = \bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

We use the *t* distribution on $(n-1)$ **degrees of freedom (df)**.

<u>Rounding Rule for *E*</u>

Round off (or up) the margin of error *E* to the **same number of decimal places** as the given value of $\bar{x}$.

$(1-\alpha)$ Confidence Interval (CI) for p

(Assume $X \sim \text{Bin}(n, p)$. To justify a **normal approximation**, verify: $n\hat{p} \ge 5$, and $n\hat{q} \ge 5$.)

$$p = \hat{p} \pm E$$

where the **margin of error**

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

That is,

$$p = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

<u>(SEE NEXT PAGE!)</u>

Determining Sample Size n for Estimating p

For a $(1-\alpha)$ **confidence level** and a desired **margin of error** E,
the **conservative** estimate for the required sample size n is given by:

$$n = \left\lceil \frac{\left(z_{\alpha/2}\right)^2 (0.25)}{E^2} \right\rceil$$

where $\lceil \quad \rceil$ is the ceiling (or "round-up") operator.

$(1-\alpha)$ Confidence Intervals (CIs) for σ^2 and σ

(Assume $X \overset{\text{approx.}}{\sim}$ Normal.)

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

We use the χ^2 distribution on $(n-1)$ **degrees of freedom (df)**.

χ_R^2 is the right (greater) CV.

χ_L^2 is the left (lesser) CV.

Rounding Rule for the Limits of the CIs

Round off (or "out") the limits of the CIs to the **same number of decimal places** as the given value of s^2 or s.