# QUIZ 1 (CHAPTERS 1-4)
# SOLUTIONS

**MATH 119 – FALL 2012 – KUNIYUKI**
**105 POINTS TOTAL, BUT 100 POINTS = 100%**
**Show all work, simplify as appropriate, and use “good form and procedure” (as in class).**
**Box in your final answers!**
**No notes or books allowed. A scientific calculator is allowed.**

1) (8 points). The Leeroy Jenkins Fan Club has 15 members. Do a stem-and-leaf plot (or stemplot) of their ages in years. Here are the ages:

45 64 32 25 38 36 51 38 61 62 22 40 57 26 43

Stage 1: Attach leaves to stems.

| | |
|---|---|
| 2 | 526 |
| 3 | 2868 |
| 4 | 503 |
| 5 | 17 |
| 6 | 412 |

Stage 2: Sort leaves for each stem.

| | |
|---|---|
| 2 | 256 |
| 3 | 2688 |
| 4 | 035 |
| 5 | 17 |
| 6 | 124 |

2) (8 points total). A random sample of homeowners in a wealthy community is taken, and they are asked how many homes they own. All of the results are summarized in the frequency table below.

| Number of homes | Frequency |
|---|---|
| 1 | 39 |
| 2 | 52 |
| 3 | 41 |
| More than 3 | 26 |

a) Find the relative frequency of homeowners in the sample who own exactly two homes. Write your answer as a decimal rounded off to three decimal places.

The sample size is: $39+52+41+26=158$.

The desired relative frequency = $\frac{52}{158} \approx \boxed{0.329}$.

b) If we were to construct a pie chart, how large should the central angle be for the pie slice labeled “2 homes”? Give your answer to the nearest degree.

$$\begin{pmatrix}\text{Relative frequency}\\ \text{of "2 homes"}\end{pmatrix}\left(360^\circ\right)=\left(\frac{52}{158}\right)\left(360^\circ\right)\approx \boxed{118^\circ}$$

Note: Avoid rounding off intermediate results. If you round off $\frac{52}{158}$, for example, either keep all the digits on your calculator, or round off to at least, say, five significant digits.

3) (33 points total). A statistics class has six students. Their scores on the final (in points) are listed below:

| | |
|---|---|
| Bart | 80 |
| Lisa | 98 |
| Martin | 100 |
| Milhouse | 80 |
| Nelson | 88 |
| Ralph | 70 |

a) (4 points). Find the mean of these scores. Show work!

$$\frac{80+98+100+80+88+70}{6}=\frac{516}{6}$$
$$=\boxed{86.0 \text{ points}}$$

b) (4 points). Find the median of these scores.

First, sort the scores in, say, ascending order.

70 80 80 88 98 100

The two middle scores are 80 and 88 points. We average them to get the median.

Answer: $\boxed{84.0 \text{ points}}$.

c) (4 points). Find the mode of these scores.

The most common score is: $\boxed{80 \text{ points}}$.

d) (4 points). Find the midrange of these scores.

$$\frac{\text{Min}+\text{Max}}{2}=\frac{70+100}{2}$$
$$=\boxed{85.0 \text{ points}}$$

e) (4 points). Find the range of these scores.

$$\text{Max} - \text{Min} = 100-70$$
$$=\boxed{30 \text{ points}}$$

f) (13 points). Find the population standard deviation of these scores. Treat the data set as a population data set, not a sample data set. Round off your final answer to one decimal place. Show all work!

From a), we know that the mean is 86.0 points. That is, $\mu = 86.0$ points.

| $x$ | $x-\mu$ | $(x-\mu)^2$ |
|---|---|---|
| 80 | $-6$ | 36 |
| 98 | 12 | 144 |
| 100 | 14 | 196 |
| 80 | $-6$ | 36 |
| 88 | 2 | 4 |
| 70 | $-16$ | 256 |
| | | Sum = 672 |

The population variance equals the average of the squared deviations in column 3.

$$\sigma^2 = \frac{\text{Sum}}{N} = \frac{672}{6} = 112 \left[\text{points}^2\right]$$

The population standard deviation equals the square root of the above.

$$\sigma = \sqrt{112} \approx \boxed{10.6 \text{ points}}$$

4) (2 points). Which of the following is more sensitive to outliers? Box in one.

$\boxed{\text{The mean}}$ The median

5) (5 points). Your grade in a class is based entirely on three midterms and a final exam. All of the exams are graded out of 100 points. You get midterm grades of 96, 82, and 67 points. You get 71 points on your final exam. Find your weighted class average if the midterms each count for 20% and the final counts for 40% of the overall grade. Do not round off; write your answer out to three significant digits. Show all work!

<u>Method 1</u>

$$\frac{(0.20)(96)+(0.20)(82)+(0.20)(67)+(0.40)(71)}{1} = \boxed{77.4 \text{ points}}$$

<u>Method 2</u>

$$\frac{(20)(96)+(20)(82)+(20)(67)+(40)(71)}{100} = \frac{7740}{100} = \boxed{77.4 \text{ points}}$$

6) (4 points total; 2 points each). A large class takes a test.

a) According to Chebyshev's Theorem, **at least** what fraction (or percent) of the scores will lie within **two** standard deviations of the mean?

If $k = 2$, then $1 - \frac{1}{k^2} = 1 - \frac{1}{(2)^2} = 1 - \frac{1}{4} = \frac{3}{4}$, which is 75%. Answer: $\boxed{\text{At least } \frac{3}{4}}$.

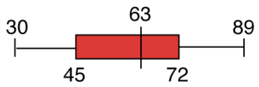
b) If we assume that the test scores have an approximately bell-shaped normal distribution, about what **percent** of the scores will lie within **two** standard deviations of the mean?

$\boxed{\text{About } 95\%}$, by the Empirical (68-95-99.7) Rule …

7) (4 points). Gremlins have a mean weight of 15 pounds, and the standard deviation of their weights is 3 pounds. Stripe, who is a Gremlin, weighs 23 pounds. Find the *z* score for Stripe's weight. Round off to two decimal places.

$z = \frac{x - \mu}{\sigma} = \frac{23 - 15}{3} \approx \boxed{2.67}$. (By the way, Stripe's weight is unusually high.)

8) (4 points total; 2 points each). The scores on a test (in points) in a large class are summarized by the boxplot (also known as a "box-and-whisker" plot) below.



a) What score (in points) was the median score on the test? $\boxed{\text{63 points.}}$

b) What score (in points) was at the first quartile, $Q_1$? $\boxed{\text{45 points.}}$

9) (4 points). All the voters in Shadyville vote for Mayor. Each voter has two choices: vote for Scum or vote for Slime. We find that 350 people voted, and 130 voted for Scum. What is the probability that a randomly selected voter in Shadyville voted for Slime?

The number of voters in Shadyville who voted for Slime is: $350 - 130 = 220$.

The desired probability is: $\boxed{\frac{220}{350} = \frac{22}{35} \approx 0.629 \text{ or } 62.9\%}$

10) (4 points total). Consider the following events:

Event A: Jack randomly picks a card and gets an Ace.
Event B: Jill flips a fair coin and gets "heads."

Are the two events **mutually exclusive** (i.e., **disjoint**)? Box in one:

Yes [✓ No]

The two events can both occur together.

Are the two events **independent**? Box in one:

[✓ Yes] No

The occurrence of one event does not affect our probability assessments of the other event.

11) (3 points). *A* and *B* are events. Fill in the blank, based on a formula we have discussed in class:

$$P(A \text{ or } B) = P(A) + P(B) - \underline{\boxed{P(A \text{ and } B)}}$$

12) (5 points). The Smiths will have five children. What is the probability that all five will be of the same gender (all male or all female)? Assume that boys and girls are equally likely and that the births are independent with respect to gender.

We want the probability that the last four children will be the same gender as the first child. (Remember the "dice doubles" and "flat tire" problems?) We assume independence among the gender identities of the five children.

$$P(\text{2nd child's gender matches the 1st}) \cdot P(\text{3rd matches 1st}) \cdot P(\text{4th matches 1st}) \cdot P(\text{5th matches 1st})$$

$$= (0.5) \cdot (0.5) \cdot (0.5) \cdot (0.5) \text{ or } \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= (0.5)^4 \text{ or } \left(\frac{1}{2}\right)^4$$

$$= \boxed{0.0625 \text{ or } \frac{1}{16} \text{ or } 6.25\%}$$

13) (7 points). Fifteen men and ten women of equal ability run in a race. A gold medal is awarded for first place, a silver medal for second place, and a bronze medal for third place. What is the probability that a man wins the gold and women win the silver and the bronze? Assume that there are no ties.

A total of 25 people run in this race.

A person cannot win more than one medal, so the three events “man wins gold,” “woman wins silver,” and “woman wins bronze” are dependent.

$$P(\text{man gold, and woman silver, and woman bronze})$$

$$= P(\text{man gold})\cdot P(\text{woman silver}\,|\,\text{man gold})\cdot P(\text{woman bronze}\,|\,(\text{man gold and woman silver}))$$

$$= \frac{15}{25}\cdot\frac{10}{24}\cdot\frac{9}{23}$$ (See my Footnote below.)

$$\begin{array}{r} = \\ \\ \approx \end{array}\boxed{\begin{array}{l} \dfrac{1350}{13{,}800} \text{ or } \dfrac{9}{92} \\ 0.0978 \text{ or } 9.78\% \end{array}}$$

Footnote:

15/25: 15 men out of the 25 runners
10/24: 10 women out of the 24 remaining runners, assuming a man wins gold
9/23: 9 women left out of the 23 remaining runners, assuming a man wins gold and a woman wins silver.

14) (4 points total; 2 points each)

a) For any two possible events $A$ and $B$, is it always true that $P(B\,|\,A) = P(A\,|\,B)$? Box in one:

Yes [✓ No]

Beware the confusion of the inverse! Refer to the notes on Section 4-5, in which we studied the cancer (or cooties) example.

b) For any two **independent** possible events $A$ and $B$, is it always true that $P(B\,|\,A) = P(B)$? Box in one:

[✓ Yes] No

Knowledge of whether $A$ occurs or not cannot have any impact on our assessment of the probability of $B$ occurring.

15) (10 points total). Each student at Adam Sandler University studies either Comedy or Drama, but not both. 327 of the female students study Comedy. 512 of the female students study Drama. 414 of the male students study Comedy. 298 of the male students study Drama. A student at the university is randomly selected. (A table will help!)

| | Comedy | Drama | |
|---|---|---|---|
| Female | 327 | 512 | **839** |
| Male | 414 | 298 | **712** |
| | **741** | **810** | **1551** |

(Marginals and Grand Total are boldfaced above.)

a) What is the probability that the student studies Comedy **or** is female?

| | Comedy | Drama | |
|---|---|---|---|
| Female | **327** | **512** | 839 |
| Male | **414** | 298 | 712 |
| | 741 | 810 | **1551** |

$$P\left(\text{Comedy or Female}\right)=\frac{\#\left(\text{Comedy or Female}\right)}{\#\left(\text{Total students}\right)}$$

$$=\frac{327+512+414}{1551}$$

$$\boxed{\begin{aligned}&=\frac{1253}{1551}\\ &\approx 0.808\ \text{ or }\ 80.8\%\end{aligned}}$$

b) Assume that we now know that the student is male. What is the probability that the student studies Drama, given that he is male?

| | Comedy | Drama | |
|---|---|---|---|
| Female | 327 | 512 | 839 |
| Male | 414 | **298** | **712** |
| | 741 | 810 | 1551 |

$$P\left(\text{Drama}\,\middle|\,\text{Male}\right)=\frac{\#\left(\text{Male and Drama}\right)}{\#\left(\text{Male}\right)}$$

$$\boxed{\begin{aligned}&=\frac{298}{712}\ \text{ or }\ \frac{149}{356}\\ &\approx 0.419\ \text{ or }\ 41.9\%\end{aligned}}$$