QUIZ 1 (CHAPTERS 1-4) SOLUTIONS

MATH 119 – SPRING 2013 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

1) (6 points). A college has 32 course sections in math. A frequency table for the numbers of students in the sections is given below. Fill out the relative frequency column. Write out the entries as decimals rounded off to three decimal places.

Number of students	Frequency	Relative Frequency
20-29	4	$\frac{4}{32} = \mathbf{0.125}$
30-39	10	$\frac{10}{32} \approx \mathbf{0.313}$
40-49	9	$\frac{9}{32} \approx \mathbf{0.281}$
50-59	6	$\frac{6}{32} \approx \mathbf{0.188}$
60-69	3	$\frac{3}{32} \approx \mathbf{0.094}$

2) (8 points). A police officer measures the speed (in mph) of 15 cars that pass by her on a highway. Do a stem-and-leaf plot (or stemplot) of the speeds (in mph). Here are the speeds:

75 54 57 47 81 51 78 84 49 61 60 67 72 90 54

Stage 1: Attach leaves to stems.

4	79
5	4714
6	107
7	582
8	14
9	0

Stage 2: Sort leaves for each stem.

4	79
5	1447
6	017
7	258
8	14
9	0

3) (33 points total). We are tracking the weekly sales of cell phones at a megastore over a five-week period. The weekly sales are listed below:

110 108 115 108 119

a) (4 points). Find the mean of these weekly sales. Show work!

$$\frac{110+108+115+108+119}{5} = \frac{560}{5} = \boxed{112 \text{ sales}}$$

b) (4 points). Find the median of these weekly sales.

First, sort the numbers in, say, ascending order.

108 108 110 115 119

The number in the middle is 110.

Answer: $\boxed{\text{110 sales}}$

c) (4 points). Find the mode of these weekly sales.

The most common number is: $\boxed{\text{108 sales}}$

d) (4 points). Find the midrange of these weekly sales.

$$\frac{\text{Min}+\text{Max}}{2}=\frac{108+119}{2}$$
$$=\boxed{113.5\text{ sales}}$$

e) (4 points). Find the range of these weekly sales.

$$\text{Max} - \text{Min} = 119-108$$
$$=\boxed{11\text{ sales}}$$

f) (13 points). Find the sample standard deviation of these weekly sales. Treat the data set as a sample data set, not a population data set. Round off your final answer to one decimal place. Show all work!

From a), we know that the sample mean is 112. That is, $\bar{x}=112$.

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
110	-2	4
108	-4	16
115	3	9
108	-4	16
119	7	49
		Sum = 94

The sample variance is the "tilted" (inflated) average of the values in column 3.

$$s^2=\frac{\text{Sum}}{n-1}=\frac{94}{4}=23.5\left[\text{sales}^2\right]$$

The sample standard deviation equals the square root of the above.

$$s=\sqrt{23.5}\approx\boxed{4.8\text{ sales}}$$

4) (3 points). Based on our discussion in class, which of the measures of spread below tends to be more sensitive to outliers? Box in one:

$\boxed{\text{The range}}$ The standard deviation

5) (6 points). Cartman's report card for last term is below.

Class	**Number of units**	**Grade**
English	4	C
Math	5	D
Physical Ed.	3	F

What was Cartman's grade point average (GPA) for last term rounded off to two decimal places? Use the point scheme discussed in class, where 4.0 represented a straight-A record, 3.0 for B, 2.0 for C, 1.0 for D, and 0.0 for F.

$$\text{GPA} = \frac{4(2)+5(1)+3(0)}{12} = \frac{13}{12} \approx \boxed{1.08 \text{ grade points}}$$

6) (4 points total; 2 points each). Let's say a large population data set is approximately normally distributed.

a) About what percent of the data will lie within one standard deviation of the mean?

$\boxed{\text{About } 68\%}$, by the Empirical (68-95-99.7) Rule.

b) About what percent of the data will lie within two standard deviations of the mean?

$\boxed{\text{About } 95\%}$, by the Empirical (68-95-99.7) Rule.

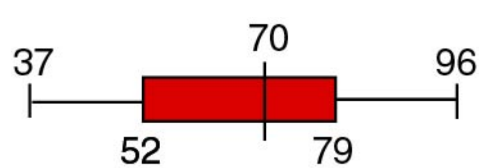
7) (4 points). According to an AAPA census, the mean number of days of paid vacation time offered for physicians' assistants is 17 days. The standard deviation is 6.6 days. If J.D. is a physician's assistant who is offered 5 days of paid vacation time, what is J.D.'s *z* score? Round off to two decimal places.

$$z = \frac{x-\mu}{\sigma} = \frac{5-17}{6.6} \approx \boxed{-1.82}$$

8) (2 points). The fourth decile (D_4) corresponds to which percentile?

$\boxed{P_{40}}$, the 40th percentile.

9) (2 points). The scores on a test (in points) in a large class are summarized by the boxplot (also known as a "box-and-whisker" plot) below. Pat scores 52 points on the test. At what percentile does Pat score?



$\boxed{P_{25}}$, the 25th percentile. This is because 52 points is Q_1, the first quartile.

10) (5 points). A and B are events such that $P(A) = 0.7$, $P(B) = 0.4$, and $P(A \text{ and } B) = 0.3$. Find $P(A \text{ or } B)$.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= 0.7 + 0.4 - 0.3 \\ &= \boxed{0.8} \end{aligned}$$

11) (4 points total; 2 points each). A standard six-sided die is rolled once. Consider the following events:

Event A: The die comes up an even number.
Event B: The die comes up a "3."

a) Are the two events <u>mutually exclusive</u> (that is, <u>disjoint</u>)? Box in one:

$\boxed{\text{Yes}}$ No

The events cannot both happen together.

b) Are the two events <u>independent</u>? Box in one:

Yes $\boxed{\text{No}}$

If, for example, the die comes up a "3," then we know that it does not come up even.

12) (5 points). The state of Denial has millions of athletes. Seventy percent of the athletes in Denial are taking the steroid Hulk. Four athletes in Denial are randomly selected. What is the probability that all four selected athletes are **not** taking Hulk?

If 70% of the athletes in Denial are taking Hulk, then 30% of the athletes in Denial are not taking Hulk; this is the complementary probability. By the Sampling Rule, we will assume independence among the selections.

$$\begin{aligned} P(\text{all 4 do not take Hulk}) &= \left[P(\text{a particular athlete does not take Hulk})\right]^4 \\ &= (0.3)^4 \text{ or } \left(\frac{3}{10}\right)^4 \\ &= \boxed{0.0081 \text{ or } \frac{81}{10{,}000}} \end{aligned}$$

13) (6 points). A red die and a green die are rolled, and a fair coin is flipped. The dice are standard six-sided dice. What is the probability that all of the following happen: the red die comes up "odd," the green die comes up a "3," and the coin comes up "heads"? (Your answer will be one number, not three.)

We assume independence.

$$P(\text{red is odd and green is "3" and coin is "heads"})$$
$$= P(\text{red is odd}) \cdot P(\text{green is "3"}) \cdot P(\text{coin is "heads"}) \quad \text{by independence}$$
$$= \frac{3}{6} \cdot \frac{1}{6} \cdot \frac{1}{2} \quad \text{or} \quad \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{2}$$
$$= \boxed{\frac{1}{24} \approx 0.0417}$$

14) (7 points). A paper bag has five blue M&Ms, four green M&Ms, and two yellow M&Ms. You randomly select three M&Ms one-by-one <u>without replacement</u> (meaning that M&Ms are <u>not</u> returned to the bag after they are taken out). What is the probability that you pick a green M&M first, a blue M&M second, and another green M&M third? (Your answer will be one number, not three.)

The total number of M&Ms in the bag is: $5+4+2=11$ M&Ms.

$$P(\text{green 1st and blue 2nd and green 3rd})$$
$$= P(\text{green 1st}) \cdot P(\text{blue 2nd} \mid \text{green 1st}) \cdot P(\text{green 3rd} \mid \text{green 1st and blue 2nd})$$
$$= \frac{4}{11} \cdot \frac{5}{10} \cdot \frac{3}{9} \quad \text{(See my Footnote below.)}$$
$$= \boxed{\frac{2}{33} \approx 0.0606 \text{ or } 6.06\%}$$

Footnote:

4/11: 4 green out of 11 M&Ms
5/10: 5 blue out of 10 remaining M&Ms, assuming a green one was picked 1st
3/9: 3 green out of 9 remaining M&Ms, assuming a green one was picked 1st and a blue one was picked 2nd

15) (10 points total). A snack foods company is conducting a taste test between Liver Chips and Crunchy Broccoli Chips. The marketers for the company are interested in, among other things, the different reactions among people of different ages. The information for the 150 people who took the taste test is in the table below. (Ages are in years.)

	Prefer Liver Chips	**Prefer Crunchy Broccoli Chips**
Age 11-30	35	5
Age 31-50	40	20
Age 51-70	20	30

Let's say we randomly select someone who took the taste test.

a) What is the probability that the person's age is in the 51-70 category **or** the person prefers Crunchy Broccoli Chips?

	Prefer Liver Chips	**Prefer Crunchy Broccoli Chips**
Age 11-30	35	**5**
Age 31-50	40	**20**
Age 51-70	**20**	**30**

$$P(\text{age 51-70 or prefers C.B. Chips}) = \frac{\#(\text{age 51-70 or prefers C.B. Chips})}{\#(\text{tasters total})}$$

$$= \frac{20+5+20+30}{150} = \frac{75}{150} = \boxed{\frac{1}{2} = 0.5 \text{ or } 50\%}$$

b) Assume that we now know that the person is in the 11-30 age range. What is the probability that the person prefers Liver Chips, **given that** the person is in the 11-30 age range?

	Prefer Liver Chips	**Prefer Crunchy Broccoli Chips**
Age 11-30	***35***	**5**
Age 31-50	40	20
Age 51-70	20	30

$$P(\text{prefers Liver Chips} \mid \text{age 11-30}) = \frac{\#(\text{age 11-30 and prefers Liver Chips})}{\#(\text{age 11-30})}$$

$$= \frac{35}{35+5} = \frac{35}{40} = \boxed{\frac{7}{8} = 0.875 \text{ or } 87.5\%}$$