QUIZ 2 (SECTION 4-6, CHAPTER 5) SOLUTIONS

MATH 119 – FALL 2012 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

1) (5 points). A math department has nine teachers. Bill Gates decides to give four of the teachers checks for $1 million, but he doesn't know which ones yet. If we only care about which teachers get the checks and which don't, how many ways can he do this?

There are ${}_9C_4 = \binom{9}{4} = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{(24) \cdot (\cancel{5!})} = \boxed{126}$ possible ways of choosing four people from a group of nine. Order is irrelevant.

2) (5 points). Bill Gates's (cheaper) rival is Gill Bates. Gill Bates also decides to give checks to four teachers in this math department of nine teachers. He will give one check for $1000, one for $500, one for $250, and one for $100. If we do care about which teachers get which checks, how many ways can he do this? (Assume that Gill doesn't care how Bill gave out his checks.)

Method 1

There are 9 choices for the $1000 check.
Then, there are 8 choices for the $500 check.
Then, there are 7 choices for the $250 check.
Then, there are 6 choices for the $100 check.
There are $9 \cdot 8 \cdot 7 \cdot 6 = \boxed{\mathbf{3024}}$ possible ways for Gill to give out his checks.

Method 2

Because we care about the "order" among the teachers who get the checks (where order is based on the dollar value of the checks), we are interested in the number of partial permutations, ${}_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{(\cancel{5!})} = \boxed{\mathbf{3024}}$ (ways).

3) (5 points). Principal Evil wants to fire all nine math teachers one-by-one. How many possible orders are there for all nine math teachers to be fired?

There are $9! = \boxed{\mathbf{362,880}}$ possible ways to order all nine teachers.

4) (5 points). You need to select a password for the SkyNet computer system. A valid SkyNet password must consist of five characters, beginning with three uppercase English letters and ending with two digits. (There are 26 uppercase English letters, and there are ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.) Those are the only restrictions. How many possible valid SkyNet passwords are there?

There are 26 possibilities for the first character, an uppercase letter; the same goes for the second character and the third character. There are 10 possibilities for the fourth character, a digit; the same goes for the fifth character. By the Multiplication Rule for Counting, there are: $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10$, or $26^3 \cdot 10^2 = \boxed{\mathbf{1,757,600}}$ valid passwords.

5) (9 points). A cooties insurance policy costs $50 for this year. If you catch cooties this year, you will receive a check for $1000. If you do not catch cooties, you will receive nothing. There is a 4% chance that you catch cooties this year. If you decide to purchase this policy, what is the expected value of your change in wealth this year as a result?

Let X = the change in your wealth as a result of purchasing the policy.

Result	***x* (in $)**	***P*(*x*)**
You catch cooties this year.	$1000 - 50 = 950$	0.04
You do not.	-50	$1 - 0.04 = 0.96$

$$E(X) = (950)(0.04) + (-50)(0.96)$$
$$= \boxed{-\$10}$$

6) (17 points total). A probability distribution is given below.

x	$P(x)$
5	0.412
10	0.579
15	0.009

a) Find the mean of this distribution. (5 points)

$$\mu = (5)(0.412) + (10)(0.579) + (15)(0.009)$$
$$= \boxed{7.985}$$

b) Find the standard deviation of this distribution. (12 points)

First, find $E(X^2)$.

x	$P(x)$	x^2
5	0.412	25
10	0.579	100
15	0.009	225

$$E(X^2) = (25)(0.412) + (100)(0.579) + (225)(0.009)$$
$$= 70.225$$

Then, find the variance.

Think: Variance = Mean of the square – Square of the mean.

$$\sigma^2 = E(X^2) - [\underbrace{E(X)}_{=\mu}]^2 = 70.225 - (7.985)^2 \approx 6.4648$$

The standard deviation is the square root of this.

$$\sigma \approx \sqrt{6.4648}$$
$$\approx \boxed{2.543}$$

7) (24 points). X is a random variable that has as its probability distribution the binomial distribution $\text{Bin}(n = 4,\ p = 0.58)$. X counts the number of successes among the four trials. Describe this distribution by filling out the table below. Show all work! When rounding off calculations, do so to at least five significant digits, except round off your answers in the table to three decimal places.

x	$P(x)$
0	**0.031**
1	**0.172**
2	**0.356**
3	**0.328**
4	**0.113**

Trial failure probability: $q = 1 - p = 1 - 0.58 = 0.42$

Binomial probability formula: $P(x) = \binom{n}{x} p^x q^{n-x} \quad (x = 0, 1, 2, \ldots, n)$,

where $n = 4$ trials.

$$P(0) = \underbrace{\binom{4}{0}}_{=1} \underbrace{(0.58)^0}_{=1} (0.42)^{4-0}$$
$$= (0.42)^4$$
$$\approx \mathbf{0.031}$$

$$P(1) = \underbrace{\binom{4}{1}}_{=4} (0.58)^1 (0.42)^{4-1}$$
$$= 4(0.58)(0.42)^3$$
$$= 4(0.58)(0.074088)$$
$$\approx \mathbf{0.172}$$

$$P(2) = \binom{4}{2} (0.58)^2 (0.42)^{4-2}$$
$$= \frac{4!}{2!2!} (0.58)^2 (0.42)^2$$
$$= 6(0.3364)(0.1764)$$
$$\approx \mathbf{0.356}$$

$$P(3) = \binom{4}{3} (0.58)^3 (0.42)^{4-3}$$
$$= \binom{4}{1} (0.58)^3 (0.42)^1$$

(by symmetry of binomial coeffs.)

$$\approx 4(0.19511)(0.42)$$
$$\approx \mathbf{0.328}$$

$$P(4) = \binom{4}{4} (0.58)^4 (0.42)^{4-4}$$
$$= \underbrace{\binom{4}{0}}_{=1} (0.58)^4 \underbrace{(0.42)^0}_{=1}$$
$$= (0.58)^4$$
$$\approx \mathbf{0.113}$$

(or subtract the other probabilities from 1)

You could also use Pascal's Triangle to get the $\binom{4}{x}$ values: 1, 4, 6, 4, 1.

Compare our results to Table A-1: $n = 4$, with $p = 0.60$ (especially) and $p = 0.50$.
Note that our probabilities add up to 1.

8) (9 points). A multiple-choice test consists of ten questions, each with four options (A, B, C, and D). Each question has exactly one correct option. Assume that a student guesses randomly on all the questions; that is, each option is equally likely to be chosen by the student on any question. What is the probability that the student gets exactly four questions correct? Show work!

Let X = the number of correct responses on the test.

Then, $X \sim \text{Bin}\left(n = 10,\ p = \frac{1}{4} \text{ or } 0.25\right)$. Also, $q = 1 - p = 1 - 0.25 = 0.75$, or $\frac{3}{4}$.

$$P(X=4) = \binom{10}{4}(0.25)^4(0.75)^{10-4} \quad \text{(Binomial Probability Formula)}$$

$$= \frac{10!}{4!6!}(0.25)^4(0.75)^6$$

$$\approx 210(0.0039063)(0.17798)$$

$$\approx \boxed{\mathbf{0.146}}$$

(Compare this to the 0.088 for n = 10, p = 0.20, x = 4 and the 0.200 for p = 0.30 from Table A-1.)

9) (10 points total). The seniors at a large high school take a standardized test, and 80% of them pass it. Some administration officials suspect that there was rampant cheating on the test at this school. Twelve seniors are randomly selected from this high school for a retest. Use Table A-1 (see the back of the test). Show work.

a) What is the probability that more than nine of the seniors taking the retest passed the test the first time? (5 points)

Let X = the number of seniors in the retest who passed the test the first time.

Then, $X \sim \text{Bin}(n = 12,\ p = 0.80)$

$$P(X > 9), \text{ or } P(\text{more than } 9) = P(10) + P(11) + P(12)$$

$$\approx 0.283 + 0.206 + 0.069 \quad \text{(from Table A-1)}$$

$$\approx \boxed{\mathbf{0.558}}$$

b) What is the probability that at least eleven of the seniors taking the retest passed the test the first time? (5 points)

$$P(X \geq 11), \text{ or } P(\text{at least } 11) = P(11) + P(12)$$

$$\approx 0.206 + 0.069 \quad \text{(from Table A-1)}$$

$$\approx \boxed{\mathbf{0.275}}$$

10) (16 points total). Senator Suspicious thinks that the news reporters in his state are biased against him, compared to the voters in his state. A trusted poll shows that 55% of the registered voters in the state support Senator Suspicious. All 360 news reporters (all of whom are registered voters) in the state are briefly interviewed. Let X represent the number of Suspicious supporters in a randomly selected group of 360 registered voters in the state. In parts a) and b), treat X as a random variable.

Then, $X \sim \text{Bin}\left(n = 360,\ p = 0.55\right)$

a) Find the mean, or expected value, of X.

$$\begin{aligned} \mu &= np \\ &= (360)(0.55) \\ &= \boxed{\textbf{198 registered voters}} \end{aligned}$$

b) Find the standard deviation of X. As usual, round off your answer to three decimal places.

$$\begin{aligned} \sigma &= \sqrt{npq} \\ &= \sqrt{(360)(0.55)(0.45)} \qquad (q = 1 - p = 1 - 0.55 = 0.45) \\ &= \sqrt{89.1} \\ &\approx \boxed{\textbf{9.439 registered voters}} \end{aligned}$$

c) In fact, out of the 360 news reporters, 195 of them say that they support Senator Suspicious. What is the corresponding z score? Round off your answer to two decimal places.

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &= \frac{195 - 198}{\sqrt{89.1}} \quad \text{or} \quad \approx \frac{195 - 198}{9.4393} \\ &\approx \boxed{\mathbf{-0.32}} \end{aligned}$$

This represents 0.32 standard deviations below the mean.

Senator Suspicious may be a bit too suspicious. The news reporters seem to be fairly representative of the overall population of registered voters in the state as far as supporting the Senator goes.