QUIZ 2 SOLUTIONS

(LESSONS 11-18: DISCRETE PROBABILITY)
MATH 119 – FALL 2019 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.
You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½.

1) (17 points). Two standard six-sided dice are rolled. One die is red; the other is green. The "total" of the two dice is the sum of the numbers on the dice. Write your answers as **exact fractions**.

- a) What is the probability of getting a 1 on the red die and a 3 on the green die? Your answer will be one fraction. (2 points)

$$\begin{aligned} &P(1 \text{ on red } \textbf{and } 3 \text{ on green}) \\ &= P(1 \text{ on red}) \cdot P(3 \text{ on green}) \quad (\text{by independence}) \\ &= \frac{1}{6} \cdot \frac{1}{6} \\ &= \boxed{\frac{1}{36}} \end{aligned}$$

Also, observe that **only one** of the 36 equally likely ordered (red, green) pairs suggested by the grid below corresponds to "1 on red **and** 3 on green."

	green	1	2	3	4	5	6
red							
1				x			
2							
3							
4							
5							
6							

- b) What is the probability that the total is 4? (5 points)

$$P(\text{total is } 4) = \boxed{\frac{3}{36}, \text{ or } \frac{1}{12}}$$

Observe that **three** of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "total is 4."

	green	1	2	3	4	5	6
red							
1				x			
2			x				
3		x					
4							
5							
6							

- c) What is the probability that the number on the green die is two more than the number on the red die? (5 points)

$$P(\text{green result} = \text{red result} + 2) = \boxed{\frac{4}{36}, \text{ or } \frac{1}{9}}$$

Observe that **four** of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "green result = red result + 2."

	green	1	2	3	4	5	6
red							
1				x			
2					x		
3						x	
4							x
5							
6							

- d) What is the probability of getting a 1 on the red die or a 3, a 4, or a 5 on the green die? Your answer will be one fraction. (5 points)

$$P(1 \text{ on red } \textbf{or } 3, 4, \text{ or } 5 \text{ on green}) = \boxed{\frac{21}{36}, \text{ or } \frac{7}{12}}$$

Observe that **21** of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "1 on red **or** 3, 4, or 5 on green." **Remember to not double-count the "x"s.**

	green	1	2	3	4	5	6
red							
1		x	x	x	x	x	x
2				x	x	x	
3				x	x	x	
4				x	x	x	
5				x	x	x	
6				x	x	x	

2) (5 points). Dum and Dee are candidates for president of Fredonia. 65% of adult Fredonians like Dum. 55% of adult Fredonians like Dee. 40% of adult Fredonians like both. What is the probability that a randomly selected adult Fredonian likes Dum or Dee? Write your answer as a decimal, a percent, or a fraction.

Use the **General Addition Rule**.

$$\begin{aligned} P(\text{likes Dum } \textbf{or } \text{likes Dee}) &= P(\text{likes Dum}) + P(\text{likes Dee}) - P(\text{likes Dum } \textbf{and } \text{likes Dee}) \\ &= 0.65 + 0.55 - 0.40 \\ &= \boxed{0.80, \text{ or } 80\%, \text{ or } \frac{80}{100}, \text{ or } \frac{4}{5}} \end{aligned}$$

3) (15 points). A beverage company conducts a taste test with 2000 people. Each person drinks some Regular Poke and informs the company whether the person likes or does not like Regular Poke. Each person drinks some Sugar-Free Poke and informs the company whether the person likes or does not like Sugar-Free Poke. Consider the following two-way frequency (or contingency) table. Write your answers as **exact fractions**.

		Likes Regular Poke? Yes	No	Total
Likes Sugar-Free Poke?	Yes	600	300	900
	No	900	200	1100
	Total	1500	500	2000

• a) What is the probability that a randomly selected person taking the taste test likes Regular Poke or Sugar-Free Poke? (5 points)

$$P(\text{likes Regular } \textbf{or} \text{ likes Sugar-Free}) = \frac{\#(\text{likes Regular } \textbf{or} \text{ likes Sugar-Free})}{N}$$

$$= \boxed{\frac{1800}{2000}, \text{ or } \frac{9}{10}}$$

For the numerator, 1800, we can either **add** the boldfaced entries below, or we could simply **subtract** 200 from 2000.

		Likes Regular Poke? Yes	No	Total
Likes Sugar-Free Poke?	Yes	**600**	**300**	900
	No	**900**	200	1100
	Total	1500	500	2000

• b) What is the **conditional probability** that a randomly selected person taking the taste test likes Regular Poke, **given that** the person likes Sugar-Free Poke? (5 points)

		Likes Regular Poke? Yes	No	Total
Likes Sugar-Free Poke?	Yes	**600**	300	**900**
	No	900	200	1100
	Total	1500	500	2000

$$P(\text{likes Regular} \mid \text{likes Sugar-Free}) = \frac{\#(\text{likes Sugar-Free } \textbf{and} \text{ likes Regular})}{\#(\text{likes Sugar-Free})}$$

$$= \boxed{\frac{600}{900}, \text{ or } \frac{2}{3}}$$

• c) What is the **conditional probability** that a randomly selected person taking the taste test likes Sugar-Free Poke, **given that** the person likes Regular Poke? (5 points)

		Likes Regular Poke? Yes	No	Total
Likes Sugar-Free Poke?	Yes	**600**	300	900
	No	900	200	1100
	Total	**1500**	500	2000

$$P(\text{likes Sugar-Free} \mid \text{likes Regular}) = \frac{\#(\text{likes Regular } \textbf{and} \text{ likes Sugar-Free})}{\#(\text{likes Regular})}$$

$$= \boxed{\frac{600}{1500}, \text{ or } \frac{2}{5}}$$

4) (4 points). A fair coin can only come up "heads" or "tails," each with the same probability. A fair coin will be flipped four times. What is the probability that the coin will come up "heads" all four times? Write your answer as an exact fraction.

$$P(4 \text{ heads in 4 flips}) = \left[P(1 \text{ head in 1 flip})\right]^4 \quad (\text{by independence}) = \left[\frac{1}{2}\right]^4 = \boxed{\frac{1}{16}}$$

5) (8 points). Three cards are randomly drawn from a standard 52-card deck **without replacement** (each drawn card is immediately, permanently removed from the deck). What is the probability that all three cards are Jacks? Write your answer as an exact fraction and also round it as a decimal to three significant figures. **Show work** by writing fractions and what you do with them!
Hint: There are originally four Jacks in the deck of cards.

Because we are drawing cards **without replacement**, previous draws affect probabilities on later draws, and the draws are **dependent** trials.

$$P(\text{Jack-1st } \textbf{and} \text{ Jack-2nd } \textbf{and} \text{ Jack-3rd})$$

$$= P(\text{Jack-1st}) \cdot P(\text{Jack-2nd} \mid \text{Jack-1st}) \cdot P(\text{Jack-3rd} \mid \text{Jack-1st and Jack-2nd})$$

$$= \underset{\text{52 cards, 4 Jacks}}{\frac{4}{52}} \cdot \underset{\text{51 cards, 3 Jacks}}{\frac{3}{51}} \cdot \underset{\text{50 cards, 2 Jacks}}{\frac{2}{50}}$$

$$= \boxed{\frac{24}{132{,}600}, \text{ or } \frac{1}{5525}}$$

$$\approx \boxed{0.000181}$$

6) (4 points). A student takes three classes. Let X = the number of classes the student will pass. The student gives the following **incomplete** probability distribution for X. Find the value of a to complete the probability distribution.

Value (x)	**Probability** $P(x)$
0	0.100
1	0.150
2	a
3	0.400

$a = 1 - 0.100 - 0.150 - 0.400 = \boxed{0.350}$.

7) (12 points). You and your family first pay \$500 for a life insurance policy for the year. If you die during the year, your family gets \$20,000. If you do not die during the year, your family gets nothing (beyond your presence, at least). Let X = you and your family's net monetary gain as a result of the decision to purchase the policy. Based on research, you assume that your probability of dying during the year is 0.020. Assuming this is correct, find $E(X)$ and **interpret** it, as we did in class. Also, fill out the table:

Outcome for the year	**Value** (x)	**Probability** $P(x)$
You live.	$\boxed{-\$500}$	$1-0.020=\boxed{0.980}$
You die.	$\$20{,}000-\$500=\boxed{\$19{,}500}$	$\boxed{0.020}$

$$\begin{aligned} E(X) \text{ or } \mu &= \sum P(x)\cdot x \\ &= (0.980)(-\$500)+(0.020)(\$19{,}500) \\ &= \boxed{-\$100} \end{aligned}$$

Over many customers in a similar situation as you, the long-run average net **loss** for each family is about \$100 per family.

8) (15 points). Showing some work or notation may help with partial credit. None of the answers is "one."

• a) You need to write four tasks on a to-do list. How many ways are there to order the four tasks in the list? (5 points)

$4! = \boxed{24}$ ways

• b) In computer science, a bit can be a "0" or a "1." A byte consists of a sequence of eight bits. How many possible bytes are there? (5 points)

$2^8 = \boxed{256}$ bytes

• c) There are five questions on a math quiz. You answer all of them. No partial credit is given; each answer is either correct or incorrect. How many ways are there for you to get three questions correct on the quiz? Assume that the order in which you answer the questions does not matter. (5 points)

${}_5C_3$, or $\binom{5}{3} = \dfrac{5!}{3!(5-3)!} = \boxed{10}$ ways

9) (5 points). A student answers all five questions on a multiple-choice math quiz. Each question has four possible options: "A," "B," "C," or "D," only one of which is correct. The student guesses randomly on all questions. The random variable is the number of questions the student gets correct. As in class, give the distribution (including the type of distribution and the values of the two parameters) that best describes the random variable.

$X \sim \boxed{\text{Bin}\left(n=5,\ p=0.25 \text{ or } \frac{1}{4}\right)}$

10) (15 points; 5 points each). It turns out that the random variable described in 9) above has the following probability distribution table.

Value (x)	**Probability** $P(x)$
0	0.237
1	0.396
2	0.264
3	0.088
4	0.015
5	0.001

Find these probabilities re the number of questions the student gets correct.

• a) P(more than 3)

$$\begin{aligned} P(X>3) &= P(4)+P(5) \\ &\approx 0.015+0.001 \\ &\approx \boxed{0.016} \end{aligned}$$

• b) P(at least 1)

$$\begin{aligned} P(X\ge 1) &= P(1)+P(2)+P(3)+P(4)+P(5) \\ &\approx 0.396+0.264+0.088+0.015+0.001 \\ &\approx \boxed{0.764} \end{aligned}$$

Easier: $P(X \ge 1) = 1 - P(0) \approx 1 - 0.237 \approx \boxed{0.763}$

• c) P(at most 2)

$$\begin{aligned} P(X\le 2) &= P(0)+P(1)+P(2) \\ &\approx 0.237+0.396+0.264 \\ &\approx \boxed{0.897} \end{aligned}$$