QUIZ 2 (SECTION 4-6, CHAPTER 5) SOLUTIONS

MATH 119 – SPRING 2013 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

1) (3 points). Which of the following is equal to ${}_nP_r$ for reasonable r and n? Box in one:

a. $\dfrac{n!}{r!}$ $\qquad$ b. $\boxed{\dfrac{n!}{(n-r)!}}$ $\qquad$ c. $\dfrac{n!}{r!(n-r)!}$

2) (3 points). Which of the following is equal to ${}_nC_r$ for reasonable r and n? Box in one:

a. $\dfrac{n!}{r!}$ $\qquad$ b. $\dfrac{n!}{(n-r)!}$ $\qquad$ c. $\boxed{\dfrac{n!}{r!(n-r)!}}$

3) (5 points). You have five different cards in your hand. How many ways are there to order the cards from left to right in your hand? "Ties" are forbidden.

There are $5! = \boxed{120}$ possible ways to order the cards.

4) (5 points). Ten teenagers want to play basketball. How many ways are there to form two teams of five people each, assuming that we do not yet care about positions on the teams? We are assigning people to "Team A" and "Team 1."

There are ${}_{10}C_5$ or $\begin{pmatrix} 10 \\ 5 \end{pmatrix} = \dfrac{10!}{5!5!} = \boxed{252}$ possible ways to construct Team A (and thus Team 1). "Order" is irrelevant in that we do not distinguish among the positions on the teams.

5) (5 points). There are nine students in Mrs. Krabappel's class. How many ways can a president, a vice president, and a treasurer be chosen? Assume that no one can hold more than one position, and there are no "ties" for the positions.

Method 1

There are 9 choices (selections) for president.
Then, there are 8 choices (selections) for vice president.
Then, there are 7 choices (selections) for treasurer.

There are $9 \cdot 8 \cdot 7 = \boxed{504}$ possible ways to select a slate for president, vice president, and treasurer.

Method 2

"Order" is relevant in the sense that we distinguish among the positions of president, vice president, and treasurer. We are interested in the number of partial permutations, ${}_9P_3 = \dfrac{9!}{(9-3)!} = \dfrac{9!}{6!} = \dfrac{9 \cdot 8 \cdot 7 \cdot \cancel{(6!)}}{\underbrace{\cancel{(6!)}}_{(1)}} = 9 \cdot 8 \cdot 7 = \boxed{504}$ possible ways.

6) (3 points). If the table below gives a probability distribution, what must the value of c be?

x	$P(x)$
0	0.102
1	0.243
2	0.500
3	c
4	0.004

The probabilities must add up to 1, so $c = 1 - 0.102 - 0.243 - 0.500 - 0.004 = \boxed{0.151}$.

7) (8 points). A game costs \$10 to play. If you win the game, you receive a check for \$100 (but you do not get your \$10 back!). The probability that a player wins the game is 0.08. You will play the game. What is the expected value of the change in your wealth?

Let X = the change in your wealth as a result of playing the game.

Result	***x* (\$)**	***P*(*x*)**
You win	$100 - 10 = 90$	0.08
You lose	-10	$1 - 0.08 = 0.92$

$$E(X) = (90)(0.08) + (-10)(0.92) = \boxed{-\$2}$$

8) (20 points total). A probability distribution is given below. In parts a) and b), round off answers to three decimal places.

x	$P(x)$
0	0.323
1	0.301
2	0.194
3	0.143
4	0.039

a) Find the mean of this distribution. (6 points)

$$\mu \text{ or } E(X) = (0)(0.323) + (1)(0.301) + (2)(0.194) + (3)(0.143) + (4)(0.039) = \boxed{1.274}$$

b) Find the standard deviation of this distribution. (14 points)

Think: Variance = Mean of the square – Square of the mean.

That is, $\sigma^2 = E(X^2) - [E(X)]^2 = E(X^2) - \mu^2$.

x	$P(x)$	x^2
0	0.323	0
1	0.301	1
2	0.194	4
3	0.143	9
4	0.039	16

$$E(X^2) = (0)(0.323) + (1)(0.301) + (4)(0.194) + (9)(0.143) + (16)(0.039) = 2.988$$

$$\sigma^2 = 2.988 - [1.274]^2 \approx 1.3649$$

$$\sigma \approx \sqrt{1.3649} \approx \boxed{1.168}$$

9) (24 points). X is a random variable that has as its probability distribution the binomial distribution $\text{Bin}(n = 4,\ p = 0.43)$. X counts the number of successes among the four trials. Describe this distribution by filling out the table below. Show all work! When rounding off calculations, do so to at least five significant digits, except round off your answers in the table to three decimal places.

x	$P(x)$
0	**0.106**
1	**0.319**
2	**0.360**
3	**0.181**
4	**0.034**

Trial failure probability $q = 1 - p = 1 - 0.43 = 0.57$.

Binomial probability formula: $P(x) = \binom{n}{x} p^x q^{n-x}$ $(x = 0, 1, 2, \ldots, n)$; $n = 4$ trials.

$$P(0) = \underbrace{\binom{4}{0}}_{=1} \underbrace{(0.43)^0}_{=1} (0.57)^{4-0}$$
$$= (0.57)^4$$
$$\approx \mathbf{0.106}$$

$$P(1) = \underbrace{\binom{4}{1}}_{=4} (0.43)^1 (0.57)^{4-1}$$
$$= 4(0.43)(0.57)^3$$
$$\approx 4(0.43)(0.18519)$$
$$\approx \mathbf{0.319}$$

$$P(2) = \binom{4}{2} (0.43)^2 (0.57)^{4-2}$$
$$= \frac{4!}{2!2!}(0.43)^2 (0.57)^2$$
$$= 6(0.1849)(0.3249)$$
$$\approx \mathbf{0.360}$$

$$P(3) = \binom{4}{3} (0.43)^3 (0.57)^{4-3}$$
$$= \binom{4}{1} (0.43)^3 (0.57)$$

(by symmetry of binomial coeffs.)

$$\approx 4(0.07951)(0.57)$$
$$\approx \mathbf{0.181}$$

$$P(4) = \binom{4}{4} (0.43)^4 (0.57)^{4-4}$$
$$= \underbrace{\binom{4}{0}}_{=1} (0.43)^4 \underbrace{(0.57)^0}_{=1}$$
$$= (0.43)^4$$
$$\approx \mathbf{0.034}$$

(or subtract the other probabilities from 1)

You could also use Pascal's triangle to get the $\binom{4}{x}$ values: 1, 4, 6, 4, 1.

Compare our results to Table A-1: $n = 4$, and $p = 0.40$ and 0.50.
Note that our probabilities add up to 1.

10) (8 points). Assume that 76% of adult Americans believe that Lindsay Lohan should get a job. Seven adult Americans are randomly selected. What is the probability that exactly five of them believe that Lindsay Lohan should get a job? Round off calculations to at least five decimal places, but round off your answer to three decimal places.

Let X = the number of the selected Americans who believe Lindsay should get a job. Then, $X \sim \text{Bin}(n=7,\ p=0.76)$. Trial failure probability $q = 1-p = 1-0.76 = 0.24$.

$$P(X=5) = \binom{n}{x} p^x q^{n-x} \ (\text{Binomial Probability Formula}) = \binom{7}{5}(0.76)^5(0.24)^{7-5}$$

$$= \frac{7!}{5!2!}(0.76)^5(0.24)^2 \approx 21(0.25355)(0.0576) \approx \boxed{0.307}$$

(Compare this to 0.318 for $n = 7$, $p = 0.70$, $x = 5$ and 0.275 for $p = 0.80$ from the table.)

11) (5 points). A test consists of nine multiple-choice questions, each with five options (A, B, C, D, and E). Each question has exactly one correct option. Assume that a student guesses randomly on all the questions; that is, each option is equally likely to be chosen by the student on any question. What is the probability that the student will get fewer than four correct answers on the test? Use Table A-1.

Let X = the number of correct answers. The probability of answering a particular question correctly is one out of five, so $p = \frac{1}{5} = 0.2$. Then, $X \sim \text{Bin}(n=9,\ p=0.2)$.

$$P(\text{fewer than } 4) = P(0)+P(1)+P(2)+P(3) \approx 0.134+0.302+0.302+0.176 \approx \boxed{0.914}$$

12) (16 points total). McWendy's has millions of regular customers. Based on a trusted survey (which we will assume to be accurate), 68% of them are satisfied with the service. Let X represent the number of service-satisfied customers in a randomly selected group of 60 regular McWendy's customers. In parts a) and b), treat X as a random variable.

Then, $X \sim \text{Bin}(n=60,\ p=0.68)$. $q = 1-p = 1-0.68 = 0.32$.

a) Find the mean of X. Don't round off.

$$\mu = np = (60)(0.68) = \boxed{40.8 \text{ customers}}$$

b) Find the standard deviation of X. Round off your answer to three decimal places.

$$\sigma = \sqrt{npq} = \sqrt{(60)(0.68)(0.32)} = \sqrt{13.056} \approx \boxed{3.613 \text{ customers}}$$

c) The McWendy's location owned by Jay and Silent Bob has 60 regular customers, and 31 of them are satisfied with the service. What is the corresponding z score? Round off your answer to two decimal places.

$$z = \frac{x-\mu}{\sigma} = \frac{31-40.8}{\sqrt{13.056}} \left(\text{or } \approx \frac{31-40.8}{3.6133}\right) \approx \boxed{-2.71}$$

This represents 2.71 standard deviations below the mean. Looks like Jay and Silent Bob's McWendy's location is offering unusually unsatisfactory service!