QUIZ 2 SOLUTIONS

(LESSONS 11-18: DISCRETE PROBABILITY)
MATH 119 – SPRING 2025 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate. You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½. Do not leave decimal parts or fractions in fractions – for example, .1/.2 or (1/3)/(2/3).

1) (15 points). Two standard six-sided dice are rolled. One die is red; the other is green. The "number" on a die is the number of holes on the side that comes up. The "total" of the two dice is the sum of the numbers on the dice. Write your answers as **exact fractions**.

- a) What is the probability of getting an even number on the red die **and** a 5 on the green die? Your answer will be one fraction. (5 points)

$$P(\text{even on red } \textbf{and } \textbf{5} \text{ on green})$$
$$= P(\text{even on red}) \cdot P(5 \text{ on green}) \quad \text{(by independence)}$$
$$= \frac{3}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{2} \cdot \frac{1}{6}$$
$$= \boxed{\frac{3}{36} \text{ or } \frac{1}{12}}$$

Also, observe that **three** of the 36 equally likely ordered (red, green) pairs suggested by the grid below corresponds to "even on red **and** 5 on green."

	green	1	2	3	4	5	6
red							
1							
2						x	
3							
4						x	
5							
6						x	

- b) What is the probability that the total is 3? Show work by using a grid as in class (you don't have to draw lines) or listing ordered pairs. (5 points)

$$P(\text{total is } 3) = \boxed{\frac{2}{36} \text{ or } \frac{1}{18}}$$

Observe that two of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "total is 3."

	green	1	2	3	4	5	6
red							
1			x				
2		x					
3							
4							
5							
6							

Ordered pairs: (1,2), (2,1).

• c) What is the probability of getting a 6 on the red die **or** a 1 or a 2 on the green die? Your answer will be one fraction. Show work by using a grid as in class (you don't have to draw lines) or listing ordered pairs.
(5 points)

$$P(\text{6 on red } \textbf{or} \text{ 1 or 2 on green}) = \boxed{\frac{16}{36} \text{ or } \frac{4}{9}}$$

Observe that **16** of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "6 on red **or** 1 or 2 on green." **Remember to not double-count the "x"s.**

Formula Bar	en	1	2	3	4	5	6
red							
1		x	x				
2		x	x				
3		x	x				
4		x	x				
5		x	x				
6		x	x	x	x	x	x

Ordered pairs: (1,1), (1,2), (2,1), (2,2), (3,1), (3,2),
(4,1), (4,2), (5,1), (5,2), **(6,1), (6,2)**,
(6,3), (6,4), (6,5), (6,6).

You could work out the 16 by taking: 6 + 6 + 6 – 2.

The idea is to add the six entries of "x"s from each of the two full columns and the full row of "x"s but then subtracting the two ordered pairs in the **overlap** that's being double-counted.

2) (5 points). Many people (the "registrants") register for a two-day conference meeting on Saturday and Sunday, but not everyone attends. 75% of the registrants attend on Saturday. 30% of the registrants attend on Sunday. 20% of the registrants attend on both Saturday and Sunday. What is the probability that a randomly selected registrant attends the conference on Saturday **or** on Sunday? Write your answer as a decimal, a percent, or a fraction. Your answer will be one number (or percent), not two numbers!

Use the **General Addition Rule**.

$$P(\text{Sat. } \textbf{or} \text{ Sun.}) = P(\text{Sat.}) + P(\text{Sun.}) - P(\text{Sat. } \textbf{and} \text{ Sun.})$$
$$= 0.75 + 0.30 - 0.20$$
$$= \boxed{0.85, \text{ or } 85\%, \text{ or } \frac{85}{100}, \text{ or } \frac{17}{20}}$$

3) (3 points). The teacher of a statistics class believes that the probability that at least one student will visit during office hours tomorrow is 0.8. What is then the probability that no students will visit during office hours tomorrow?

Let X = the number of students who will visit during office hours tomorrow.

$$P(X=0) = 1 - P(\text{at least 1}) = 1 - 0.8 = \boxed{0.2}$$

4) (20 points). The 900 registered voters in a small town are asked if they support (or do not support) the bailout of Silicon Valley Bank. Each voter in the town is either a registered Democrat or a registered Republican. Consider the following two-way frequency (or contingency) table. Write your answers as **exact fractions**. (This is based on a YouGov poll from March 13-14, 2023.)

		Party		
		Democrat	Republican	Total
Position	Support	316	310	626
	Do Not Support	84	190	274
	Total	400	500	900

Note: The YouGov poll had 10,041 U.S. adults as respondents. 79% of Democrats supported the bailout, while only 62% of Republicans supported the bailout.

- a) What is the probability that a randomly selected registered voter in the town is a Democrat **or** is a voter who supports the bailout? Your answer will be one fraction. (5 points)

		Party		
		Democrat	Republican	Total
Position	Support	**316**	**310**	626
	Do Not Support	**84**	190	274
	Total	400	500	**900**

$$P(\text{Democrat } \textbf{or} \text{ Support}) = \frac{\#(\text{Democrat } \textbf{or} \text{ Support})}{N}$$

$$= \boxed{\frac{\mathbf{710}}{\mathbf{900}} \text{ or } \frac{71}{90}}$$

For the first numerator, 710, we can either **add** the **boldfaced red** entries, or we could simply **subtract** 190 from 900.

- b) What is the probability that a randomly selected registered voter in the town is a Democrat **and** is a voter who supports the bailout? Your answer will be one fraction. (5 points)

		Party		
		Democrat	Republican	Total
Position	Support	**316**	310	626
	Do Not Support	84	190	274
	Total	400	500	**900**

$$P(\text{Democrat } \textbf{and} \text{ Support}) = \frac{\#(\text{Democrat } \textbf{and} \text{ Support})}{N}$$

$$= \boxed{\frac{\mathbf{316}}{\mathbf{900}} \text{ or } \frac{79}{225}}$$

• c) What is the **conditional probability** that a randomly selected registered voter in the town is a Republican, **given that** the voter supports the bailout? (5 points)

		Party		
		Democrat	Republican	Total
Position	Support	316	**310**	**626**
	Do Not Support	84	190	274
	Total	400	500	900

$$P(\text{ Republican} \mid \text{Support}) = \frac{\#(\text{Support } \textbf{and} \text{ Republican})}{\#(\text{Support})}$$

$$= \boxed{\frac{\mathbf{310}}{\mathbf{626}} \text{ or } \frac{155}{313}}$$

• d) What is the **conditional probability** that a randomly selected registered voter in the town supports the bailout, **given that** the voter is a Republican? (5 points)

		Party		
		Democrat	Republican	Total
Position	Support	316	**310**	626
	Do Not Support	84	190	274
	Total	400	**500**	900

$$P(\text{ Support} \mid \text{Republican}) = \frac{\#(\text{Republican } \textbf{and} \text{ Support})}{\#(\text{Republican})}$$

$$= \boxed{\frac{\mathbf{310}}{\mathbf{500}} \text{ or } \frac{31}{50}}$$

5) (13 points). A standard 52-card deck has 26 black cards and 26 red cards.

• a) Three cards are randomly drawn from a standard 52-card deck **with replacement** (each drawn card is immediately returned to the deck). What is the probability that all three cards are black? Write your answer as an exact fraction. **Show work** by showing how you got your answer! (5 points)

$$P(\text{all 3 cards are black}) = [P(\text{black card})]^3 \text{ (by independence)} = \left[\frac{26}{52}\right]^3 \text{ or } \left[\frac{1}{2}\right]^3 = \boxed{\frac{17{,}576}{140{,}608} \text{ or } \frac{1}{8}}$$

• b) Three cards are randomly drawn from a standard 52-card deck **without replacement** (each drawn card is immediately, permanently removed from the deck). What is the probability that all three cards are black? Write your answer as an exact fraction. **Show work** by writing fractions and what you do with them! (8 points)

Because we are drawing cards **without replacement**, previous draws affect probabilities on later draws, and the draws are **dependent** trials.

$$P(\text{Black}-1\text{st } \textbf{and } \text{Black}-2\text{nd } \textbf{and } \text{Black}-3\text{rd})$$
$$= P(\text{Black}-1\text{st}) \cdot P(\text{ Black}-2\text{nd} \mid \text{Black}-1\text{st}) \cdot P(\text{ Black}-3\text{rd} \mid \text{Black}-1\text{st and Black}-2\text{nd})$$
$$= \frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50}$$

52 cards, 26 black; 51 cards, 25 black; 50 cards, 24 black

$$= \boxed{\frac{15{,}600}{132{,}600}, \text{ or } \frac{2}{17}}$$

6) (9 points). Statistics teachers teach a class with five students every semester for many years. The following probability distribution is assigned for *X*, the number of students who will visit during office hours on the day of the final.

Value (x)	**Probability** $P(x)$
0	0.035
1	0.255
2	0.400
3	0.155
4	0.100
5	0.055

Find $E(X)$ and **interpret** it, as we did in class. Assume that the distribution is the same for all the classes and that it does not change over time. Hint: Consider the day of the final for many of these classes.

$$E(X) \text{ or } \mu = \sum x \cdot P(x)$$
$$= (0)(0.035) + (1)(0.255) + (2)(0.400) + (3)(0.155) + (4)(0.100) + (5)(0.055)$$
$$= \boxed{2.195 \text{ students}}$$

Over many of these classes, the **long-run average** number of students who will visit during office hours on the day of the final is about 2.195 students per class.

7) (15 points). Showing some work or notation may help with partial credit. None of the answers is "one." Each answer is a single number (of ways).

- a) How many ways are there to arrange eight different pies in a row from left to right? (5 points)

  $8! = \boxed{40{,}320}$ ways

- b) How many ways can a customer select three of the eight pies, assuming that order does not matter? (5 points)

  ${}_8C_3$, or $\binom{8}{3} = \dfrac{8!}{3!(8-3)!} = \boxed{56}$ ways

- c) A store sells six different fruit pies, four different meat pies, and three different chocolate pies. How many ways can you buy three pies at the store if one must be a fruit pie, one must be a meat pie, and one must be a chocolate pie? Assume that you only care about which fruit pie you get, which meat pie you get, and which chocolate pie you get. (5 points)

  $6 \cdot 4 \cdot 3 = \boxed{72}$ ways

8) (5 points). A student answers all seven questions on a true-false math quiz. Each question has two possible options: "true" or "false," only one of which is correct. The student guesses randomly on all questions. The random variable is the number of questions the student gets correct. As in class, give the distribution (including the type of distribution and the values of the two parameters) that best describes the random variable.

$$X \sim \boxed{\text{Bin}\left(n = 7,\ p = \frac{1}{2}\right)}$$

9) (15 points; 5 points each). It turns out that the random variable described in 8) above has the following probability distribution table.

Value (x)	**Probability** $P(x)$
0	0.008
1	0.055
2	0.164
3	0.273
4	0.273
5	0.164
6	0.055
7	0.008

Find the indicated probabilities regarding the number of questions the student gets correct. Write your answers to three decimal places. Showing work can help with partial credit; for example, rewriting using an inequality or rewriting as a sum or difference of probabilities.

- a) P(more than 5)

$$\begin{aligned} P(X>5) &= P(6)+P(7) \\ &\approx 0.055+0.008 \\ &\approx \boxed{0.063} \end{aligned}$$

- b) P(at least 5)

$$\begin{aligned} P(X\geq 5) &= P(5)+P(6)+P(7) \\ &\approx 0.164+0.055+0.008 \\ &\approx \boxed{0.227} \end{aligned}$$

- c) P(at most 1)

$$\begin{aligned} P(X\leq 1) &= P(0)+P(1) \\ &\approx 0.008+0.055 \\ &\approx \boxed{0.063} \end{aligned}$$

Note that this is the same as the answer to a) by symmetry.