QUIZ 2 SOLUTIONS

(LESSONS 11-18: DISCRETE PROBABILITY)
STAT C1000 – SPRING 2026 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate. You do <u>not</u> have to reduce fractions. For example, 10/20 does <u>not</u> have to be rewritten as ½. Do <u>not</u> leave decimal parts in fractions or fractions in fractions, such as .1/.2 or (1/3)/(2/3).

1) (12 points). Two standard six-sided dice are rolled. One die is red; the other is green. The "number" on a die is the number of holes on the side that comes up. Write your answers as **exact fractions**.

- a) What is the probability of getting a 6 on the red die? (2 points)

$P(6 \text{ on red}) = \boxed{\frac{1}{6}}$ because this is really a one-die problem.

red	1	2	3	4	5	6
						x

As a two-dice problem, we could say: $P(6 \text{ on red}) = \boxed{\frac{6}{36}, \text{ or } \frac{1}{6}}$.

Observe that **six** of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "6 on red."

	green	1	2	3	4	5	6
red							
1							
2							
3							
4							
5							
6		x	x	x	x	x	x

- b) What is the probability that the total is 9? The "total" of the two dice is the sum of the numbers on the dice. Show work by using a grid as in class (you don't have to draw lines) or listing ordered pairs. (5 points)

$$P(\text{total is } 9) = \boxed{\frac{4}{36}, \text{ or } \frac{1}{9}}$$

Observe that **four** of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "total is 9."

	green	1	2	3	4	5	6
red							
1							
2							
3							x
4						x	
5					x		
6				x			

• c) What is the probability of getting doubles or a 4 on the red die? Your answer will be one fraction. You get "doubles" when the red die and the green die come up the same number. Show work by using a grid as in class (you don't have to draw lines) or listing ordered pairs. (5 points)

$$P(\text{doubles or 4 on red}) = \boxed{\frac{11}{36}}$$

Observe that **11** of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "doubles or 4 on red." **Remember to not double-count the "x".**

	green	1	2	3	4	5	6
red							
1		x					
2			x				
3				x			
4		x	x	x	x	x	x
5						x	
6							x

2) (5 points). A school band is choosing the colors on a new uniform. 30% of the band members want orange to be on the uniform. 20% of the band members want purple to be on the uniform. 5% of the band members want both orange and purple to be on the uniform. What is the probability that a randomly selected band member wants orange or purple to be on the uniform? Write your answer as a decimal, a percent, or a fraction. Your answer will be one number (or percent), not two numbers!

Use the **General Addition Rule**.

$$\begin{aligned} P(\text{orange } \textbf{or} \text{ purple}) &= P(\text{orange}) + P(\text{purple}) - P(\text{orange } \textbf{and} \text{ purple}) \\ &= 0.30 + 0.20 - 0.05 \\ &= \boxed{0.45, \text{ or } 45\%, \text{ or } \frac{45}{100}, \text{ or } \frac{9}{20}} \end{aligned}$$

3) (20 points). Each adult in Voterville is a Democrat or a Republican (but not both). All 5000 adults in Voterville vote on a local proposition. Each adult votes "Yes" or "No" (but not both) and honestly tells the local newspaper how they voted. Consider the following two-way frequency (or contingency) table. Write your answers as **exact fractions**.

		Vote Yes	No	Total
Party	Democrat	1500	500	2000
	Republican	1250	1750	3000
	Total	2750	2250	5000

• a) An adult in Voterville is randomly selected. What is the probability that the adult is a Democrat **and** the adult voted "Yes"? Your answer will be one fraction. (5 points)

$$P(\text{Democrat } \textbf{and} \text{ "Yes"}) = \frac{\#(\text{Democrat } \textbf{and} \text{ "Yes"})}{N}$$

$$= \boxed{\frac{1500}{5000}, \text{ or } \frac{3}{10}}$$

		Vote Yes	No	Total
Party	Democrat	**1500**	500	2000
	Republican	1250	1750	3000
	Total	2750	2250	**5000**

• b) An adult in Voterville is randomly selected. What is the probability that the adult is a Democrat **or** the adult voted "Yes"? Your answer will be one fraction. (5 points)

$$P(\text{Democrat } \textbf{or} \text{ "Yes"}) = \frac{\#(\text{Democrat } \textbf{or} \text{ "Yes"})}{N}$$

$$= \boxed{\frac{3250}{5000}, \text{ or } \frac{13}{20}}$$

For the numerator, 3250, we can either **add** the **red** boldfaced entries below, or we could simply **subtract** 1750 from 5000.

		Vote Yes	No	Total
Party	Democrat	**1500**	**500**	2000
	Republican	**1250**	1750	3000
	Total	2750	2250	**5000**

• c) What is the **conditional probability** that a randomly selected adult in Voterville voted "Yes," **given that** the adult is a Republican? (5 points)

$$P(\text{"Yes"} \mid \text{Republican}) = \frac{\#(\text{Republican } \textbf{and} \text{ "Yes"})}{\#(\text{Republican})}$$
$$= \boxed{\frac{1250}{3000}, \text{ or } \frac{5}{12}}$$

		Vote		
		Yes	No	Total
Party	Democrat	1500	500	2000
	Republican	**1250**	1750	**3000**
	Total	2750	2250	5000

• d) What is the **conditional probability** that a randomly selected adult in Voterville is a Republican, **given that** the adult voted "Yes"? (5 points)

$$P(\text{Republican} \mid \text{"Yes"}) = \frac{\#(\text{"Yes" } \textbf{and} \text{ Republican})}{\#(\text{"Yes"})}$$
$$= \boxed{\frac{1250}{2750}, \text{ or } \frac{5}{11}}$$

		Vote		
		Yes	No	Total
Party	Democrat	1500	500	2000
	Republican	**1250**	1750	3000
	Total	**2750**	2250	5000

4) (15 points). A standard 52-card deck has 13 hearts, 13 diamonds, 13 clubs, and 13 spades.

• a) Four cards are randomly drawn from a standard 52-card deck **with replacement** (each drawn card is immediately returned to the deck). What is the probability that all four cards are spades? Write your answer as an exact fraction and also round it as a decimal to three significant figures. **Show work** by showing how you got your answer! (5 points)

$$P(4 \text{ spades}) = [P(\text{spade in one draw})]^4 \text{ (by independence)} = \left[\frac{1}{4}\right]^4 = \boxed{\frac{1}{256} \approx 0.00391}$$

You could also do:

$$P(4 \text{ spades}) = [P(\text{spade in one draw})]^4 = \left[\frac{13}{52}\right]^4 = \boxed{\frac{28{,}561}{7{,}311{,}616} \approx 0.00391}$$

• b) Four cards are randomly drawn from a standard 52-card deck **without replacement** (each drawn card is immediately, permanently removed from the deck). What is the probability that the first two cards are spades and the last two cards are clubs? Write your answer as an exact fraction and also round it as a decimal to three significant figures. **Show work** by writing fractions and what you do with them! (10 points)

Because we are drawing cards **without replacement**, previous draws affect probabilities on later draws, and the draws are **dependent** trials.

Let "S" = Spades and "C" = Clubs.

$P(\text{S 1st } \textbf{and} \text{ S 2nd } \textbf{and} \text{ C 3rd } \textbf{and} \text{ C 4th})$

$= P(\text{S 1st}) \cdot P(\text{S 2nd} \mid \text{S 1st}) \cdot P(\text{C 3rd} \mid \text{S 1st and S 2nd}) \cdot P(\text{C 4th} \mid \text{S 1st and S 2nd and C 3rd})$

$= \dfrac{13}{52} \cdot \dfrac{12}{51} \cdot \dfrac{13}{50} \cdot \dfrac{12}{49}$

52 cards	51 cards	50 cards	49 cards
13 spades	12 spades	13 clubs	12 clubs

$= \boxed{\dfrac{24{,}336}{6{,}497{,}400}, \text{ or } \dfrac{78}{20825}}$

$\approx \boxed{0.00375}$

5) (3 points). A mechanic is examining a car. The probability that none of the tires will need to be replaced in the next year is 0.9. What is then the probability that at least one of the tires will need to be replaced in the next year?

Let X = the number of tires that will need to be replaced in the next year.

$P(X \geq 1) = 1 - P(0) = 1 - 0.9 = \boxed{0.1}$

6) (10 points). The mechanic examines another car and assigns the following probability distribution for X, the number of tires on the car that will need to be replaced in the next year.

Value (x)	**Probability** $P(x)$
0	0.650
1	0.200
2	0.110
3	0.025
4	0.015

Find $E(X)$ and **interpret** it. Hint: Consider many cars in the same condition.

$$\begin{aligned} E(X) \text{ or } \mu &= \sum P(x)\cdot x \\ &= (0)(0.650)+(1)(0.200)+(2)(0.110)+(3)(0.025)+(4)(0.015) \\ &= \boxed{0.555 \text{ tires}} \end{aligned}$$

Over many cars in the same condition, the long-run average number of tires that will need to be replaced in the next year is 0.555 tires.

7) (15 points). Showing some work or notation may help with partial credit. None of the answers is "one." Each answer is a single number (of ways).

• a) You need to write four tasks on a to-do list. How many ways are there to order the four tasks in the list? (5 points)

$4! = \boxed{24}$ ways

• b) In computer science, a bit can be a "0" or a "1." A byte consists of a sequence of eight bits. How many possible bytes are there? (5 points)

$2^8 = \boxed{256}$ bytes

• c) There are five questions on a math quiz. You answer all of them. No partial credit is given; each answer is either correct or incorrect. How many ways are there for you to get three questions correct on the quiz? Assume that the order in which you answer the questions does not matter. (5 points)

$${}_5C_3, \text{ or } \binom{5}{3} = \frac{5!}{3!(5-3)!} = \boxed{10} \text{ ways}$$

8) (5 points). According to a Pew Research Poll from March 2022, 69% of U.S. adults favored admitting thousands of Ukrainian refugees into the U.S. (Assume that this poll is accurate.) Six U.S. adults were randomly selected for interviews. The random variable is the number of interviewed adults who favored admitting thousands of Ukrainian refugees into the U.S. As in class, give the distribution (including the type of distribution and the values of the two parameters) that best describes the random variable.

$$X \sim \boxed{\text{Bin}(n = 6,\ p = 0.69)}$$

9) (15 points; 5 points each). It turns out that the random variable described in 8) above has the following probability distribution table, with probabilities rounded off to three decimal places.

Value (x)	**Probability** $P\,(x)$
0	0.000
1	0.012
2	0.066
3	0.196
4	0.327
5	0.291
6	0.108

We could write: $P(0) = 0+$, but don't worry about that.

Find the indicated probabilities regarding the number of interviewed adults who favor admitting thousands of Ukrainian refugees into the U.S. Showing work can help with partial credit; for example, rewriting using an inequality or rewriting as a sum or difference of probabilities.

- a) $P(\text{at most } 3)$

$$\begin{aligned} P(X \le 3) &= P(0) + P(1) + P(2) + P(3) \\ &\approx 0.000 + 0.012 + 0.066 + 0.196 \\ &\approx \boxed{0.274} \end{aligned}$$

- b) $P(\text{more than } 4)$

$$\begin{aligned} P(X > 4) &= P(5) + P(6) \\ &\approx 0.291 + 0.108 \\ &\approx \boxed{0.399} \end{aligned}$$

- c) $P(\text{at least } 3)$

$$\begin{aligned} P(X \geq 3) &= P(3) + P(4) + P(5) + P(6) \\ &\approx 0.196 + 0.327 + 0.291 + 0.108 \\ &\approx \boxed{0.922} \end{aligned}$$