QUIZ 3 (CHAPTER 6) - SOLUTIONS

MATH 119 – FALL 2012 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

1) (2 points). What is the mean of the standard normal distribution? **0**.

2) (2 points). What is the standard deviation of the standard normal distribution? **1**.

3) (6 points total). The test scores of the students in a class have a mean of 50 points and a standard deviation of 10 points.

   a) If Student X scored 70 points on the test, what is that student's z score?

   2 or 2.00, because 70 points is two standard deviations above the mean.

$$z = \frac{x-\mu}{\sigma} = \frac{70-50}{10} = 2.00$$

   b) If Student Y has a z score of 3.00, what is that student's test score?

   80 points is 3.00 standard deviations above the mean.

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ 3 &= \frac{x-50}{10} \\ 30 &= x-50 \\ 80 &= x \\ x &= 80 \text{ points} \end{aligned} \qquad \textbf{or} \qquad \begin{aligned} x &= \mu + z\sigma \\ &= 50 + (3)(10) \\ &= 80 \text{ points} \end{aligned}$$

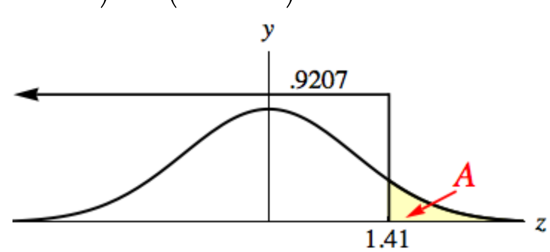
4) (38 points total). The test scores for a very large lecture class are approximately normally distributed with a mean of 62.5 points and a standard deviation of 12.4 points. Do not use continuity corrections for these problems.

   a) What percent of the test scores are higher than 80.0 points? These people will get "A"s. Write your answer to the nearest hundredth (that is, to two decimal places) of a percent. (10 points)

   Let X be a randomly selected test score (in points) in the class.

   If $x = 80.0$, then: $z = \frac{x-\mu}{\sigma} = \frac{80.0-62.5}{12.4} = \frac{17.5}{12.4} \approx 1.41$

   Therefore, $P(X > 80.0) \approx P(Z > 1.41)$.



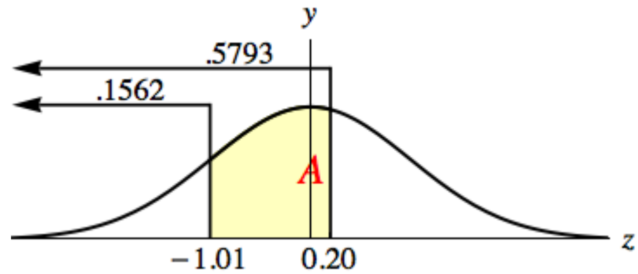
$A \approx 1 - .9207 \approx .0793 \approx$ **7.93%**

b) What <u>percent</u> of the test scores are between 50.0 points and 65.0 points? These people will get "C"s. Write your answer to the nearest hundredth of a percent. (16 points)

$$\text{If } x = 50.0\text{, then: } z = \frac{x-\mu}{\sigma} = \frac{50.0-62.5}{12.4} = \frac{-12.5}{12.4} \approx -1.01$$

$$\text{If } x = 65.0\text{, then: } z = \frac{x-\mu}{\sigma} = \frac{65.0-62.5}{12.4} = \frac{2.5}{12.4} \approx 0.20$$

Therefore, $P(50.0 \text{ points} < X < 65.0 \text{ points}) \approx P(-1.01 < Z < 0.20)$

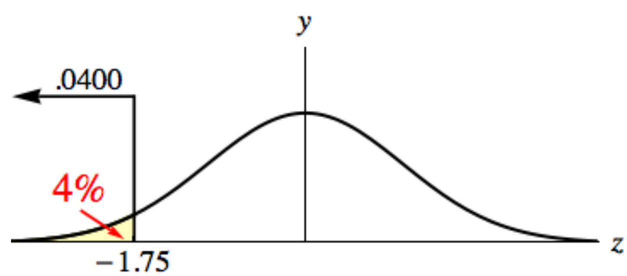


$A \approx .5793 - .1562 \approx .4231 \approx$ **42.31%**

c) The students who scored in the bottom 4% of the class will get "F"s, and only those students. Find the cutoff score for an "F." Round off your answer to the nearest point. (12 points)

We want the score that separates the bottom 4% from the top 96%.

Look in the body of Table A-2 to find the probability (or area) closest to 0.0400. The closest probability is 0.0401, and the corresponding z score is -1.75.



Find the corresponding x score:

$$x = \mu + z\sigma \approx 62.5 + (-1.75)(12.4) \approx \boxed{\textbf{41 points}}$$

You can also solve the following for x:

$$z = \frac{x-\mu}{\sigma}$$

$$-1.75 \approx \frac{x-62.5}{12.4}$$

$$(-1.75)(12.4) \approx x - 62.5$$

$$62.5 + (-1.75)(12.4) \approx x$$

$$x \approx \boxed{\textbf{41 points}}$$

5) (34 points total). The numbers of Krusty-O bits in Krusty-O cereal boxes are approximately normally distributed with mean 345.0 cereal bits and standard deviation 7.3 bits. Do not use continuity corrections for these problems.

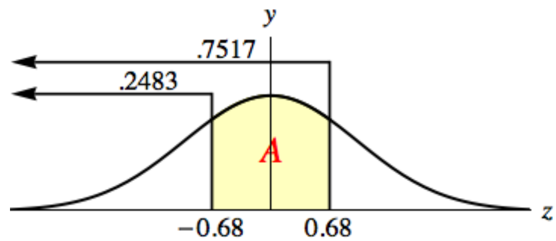
a) Find the probability that a randomly selected Krusty-O cereal box has between 340.0 and 350.0 cereal bits. (15 points)

Let X be the number of Krusty-O bits in a random box of Krusty-O cereal.

If $x = 350.0$, then: $z = \dfrac{x-\mu}{\sigma} = \dfrac{350.0 - 345.0}{7.3} = \dfrac{5.0}{7.3} \approx 0.68$

If $x = 340.0$, then: $z \approx -0.68$ by symmetry about the mean.

Therefore, $P(340.0 \text{ bits} < X < 350.0 \text{ bits}) \approx P(-0.68 < Z < 0.68)$.



$A \approx .7517 - .2483 \approx \boxed{\mathbf{.5034}}$

b) If 14 Krusty-O cereal boxes are randomly selected, what is the probability that their average number of cereal bits per box is between 340.0 bits and 350.0 bits? (19 points)

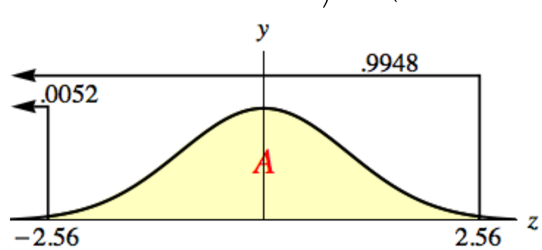
Let $\overline{X}$ be the mean number of cereal bits in a group of 14 randomly selected Krusty-O cereal boxes. We can apply the Central Limit Theorem (CLT), because the original distribution is approximately normal.

$$\overline{X} \overset{\text{approx.}}{\sim} N\left(\mu_{\overline{X}} = \mu = 345.0 \text{ bits},\ \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{7.3}{\sqrt{14}} \approx 1.951 \text{ bits}\right)$$

If $\overline{x} = 350.0$, then: $z = \dfrac{\overline{x} - \mu_{\overline{X}}}{\sigma_{\overline{X}}}$ or $\dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}} \approx \dfrac{350.0 - 345.0}{1.951} = \dfrac{5.0}{1.951} \approx 2.56$

If $\overline{x} = 340.0$, then: $z \approx -2.56$ by symmetry about the mean.

Therefore, $P(340.0 \text{ bits} < \overline{X} < 350.0 \text{ bits}) \approx P(-2.56 < Z < 2.56)$.



$A \approx .9948 - .0052 \approx \boxed{\mathbf{.9896}}$

Note: Observe that this is higher than the .5034 we obtained in part a); this is due to the tighter clustering of the $\overline{X}$ distribution about the mean in part b).

6) (23 points). According to a poll, about 37% of registered voters in a state approve of Senator Smith. Let's say 250 registered voters in the state are randomly selected. Based on the poll results, find the probability that fewer than 75 of those selected voters approve of Senator Smith. Use a normal approximation to a binomial distribution, and use a continuity correction. Show why the normal approximation is appropriate based on the rules given in class.

- Define "a success on one trial" as "the registered voter who represents the trial approves of Senator Smith."

- Then, $p = 0.37$, and $q = 1 - p = 0.63$.

- If we define X as the number of the selected registered voters who approve of Senator Smith, then $X \sim \text{Bin}(n = 250,\ p = 0.37)$.

- Show why the normal approximation is appropriate:

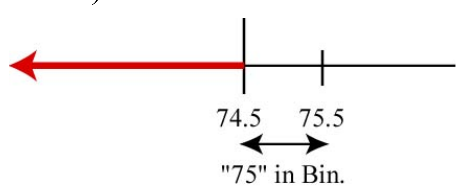
$$np = (250)(0.37) = 92.5 \geq 5 \quad (\text{Good!})$$
$$nq = (250)(0.63) = 157.5 \geq 5 \quad (\text{Good!})$$

- Therefore, we can use the approximation:

$$X \overset{\text{approx.}}{\sim} N\left(\begin{array}{l} \mu = np = 92.5 \text{ voters}, \\ \sigma = \sqrt{npq} = \sqrt{(250)(0.37)(0.63)} = \sqrt{58.275} \approx 7.634 \text{ voters} \end{array} \right)$$

- Apply the continuity correction and go to z scores:

The whole number "75" from the binomial distribution corresponds to the real numbers from 74.5 to 75.5 from the normal distribution. Since "fewer than" excludes 75, itself, we exclude those numbers but include the numbers to the left of (that is, lesser than) 74.5.

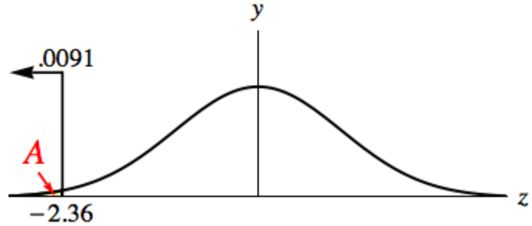


Therefore, $P(X < 75) \approx P(X_C < 74.5)$.

If $x_C = 74.5$, then: $z \approx \dfrac{74.5 - 92.5}{7.634} = \dfrac{-18}{7.634} \approx -2.36$

Therefore, $P(X < 75) \approx P(Z < -2.36)$.

- Find the desired probability:



$A \approx$ **.0091**