QUIZ 3 (CHAPTER 6) - SOLUTIONS

MATH 119 – SPRING 2013 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

1) (2 points). What is the mean of the standard normal distribution? $\boxed{0}$.
2) (2 points). What is the standard deviation of the standard normal distribution? $\boxed{1}$.
3) (2 points). One million people have taken a particular standardized test. The distribution of test scores has mean 75 points and standard deviation 6 points. We will draw a random sample of 50 people who took the test and analyze their scores. Which of the following statements is true, based on the Central Limit Theorem (CLT)? Box in one:

   [✓ i. The probability distribution for the average test score for the 50 sampled people will be approximately normally distributed with a mean of 75 points.]

   ii. The probability distribution for the average test score for the 50 sampled people will be approximately normally distributed with a standard deviation of 6 points.

4) (2 points). Based on our discussion in class, is it true that all binomial distributions can be approximated well by normal distributions? Box in one:

   Yes [✓ No]

   A normal approximation is only appropriate for a binomial distribution if $np \geq 5$ and $nq \geq 5$.

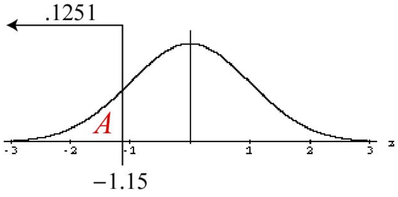
5) (38 points total). The starting salaries of chemical engineers in a state are approximately normally distributed with a mean of \$58,500 and a standard deviation of \$7,400. Do not use continuity corrections for these problems.

   a) What percent of the starting salaries are less than \$50,000? Write your answer to the nearest hundredth (that is, to two decimal places) of a percent. (8 points)

   Let X be a randomly selected starting salary of a chemical engineer in the state.

   If x = \$50,000, then: $z = \dfrac{x-\mu}{\sigma} = \dfrac{50{,}000 - 58{,}500}{7{,}400} = \dfrac{-8{,}500}{7{,}400} \approx -1.15$

   Therefore, $P\left(X < \$50{,}000\right) \approx P\left(Z < -1.15\right)$.



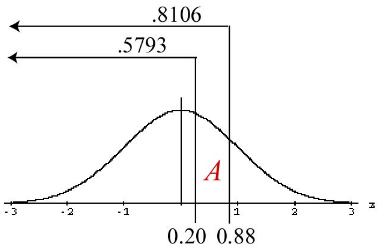
$A \approx .1251 \approx \boxed{12.51\%}$

b) What percent of the starting salaries are between \$60,000 and \$65,000? Write your answer to the nearest hundredth of a percent. (16 points)

$$\text{If } x = \$60{,}000\text{, then: } z = \frac{x-\mu}{\sigma} = \frac{60{,}000 - 58{,}500}{7{,}400} = \frac{1{,}500}{7{,}400} \approx 0.20$$

$$\text{If } x = \$65{,}000\text{, then: } z = \frac{x-\mu}{\sigma} = \frac{65{,}000 - 58{,}500}{7{,}400} = \frac{6{,}500}{7{,}400} \approx 0.88$$

Therefore, $P(\$60{,}000 < X < \$65{,}000) \approx P(0.20 < Z < 0.88)$

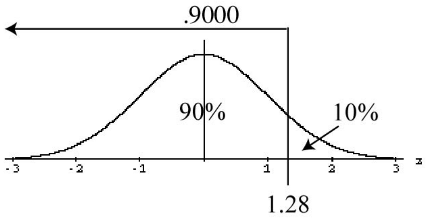


$$A \approx .8106 - .5793 \approx .2313 \approx \boxed{23.13\%}$$

c) A college claims that its chemical engineering graduates have starting salaries in the top 10% of the state's starting chemical engineering salaries. Among the state's starting chemical engineering salaries, find the salary that separates the top 10% from the rest. Write your answer to the nearest dollar. (14 points)

We want the score that separates the bottom 90% from the top 10%.

Look in the body of Table A-2 to find the probability (or area) closest to 0.9000. The closest probability is 0.8997, and the corresponding z score is 1.28.



Find the corresponding x score:

$$x = \mu + z\sigma \approx 58{,}500 + (1.28)(7{,}400) \approx \boxed{\$67{,}972}$$

You can also solve the following for x:

$$z = \frac{x-\mu}{\sigma} \Rightarrow 1.28 \approx \frac{x - 58{,}500}{7{,}400} \Rightarrow (1.28)(7{,}400) \approx x - 58{,}500 \Rightarrow$$

$$58{,}500 + (1.28)(7{,}400) \approx x \Rightarrow x \approx \boxed{\$67{,}972}$$

6) (34 points total). The heights of adult males are approximately normally distributed with a mean of 69.0 inches and a standard deviation of 2.8 inches. Do <u>not</u> use continuity corrections for these problems.

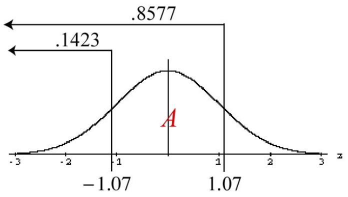
a) Find the probability that a randomly selected adult male is between 66.0 inches and 72.0 inches in height. (15 points)

Let X be the height of a randomly selected adult male (in inches).

If $x = 72.0$ in, then: $z = \dfrac{x-\mu}{\sigma} = \dfrac{72.0-69.0}{2.8} = \dfrac{3.0}{2.8} \approx 1.07$

If $x = 66.0$ in, then: $z \approx -1.07$ by symmetry about the mean.

Therefore, $P\left(66.0 \text{ in} < X < 72.0 \text{ in}\right) \approx P\left(-1.07 < Z < 1.07\right)$.



$A \approx .8577 - .1423 \approx \boxed{.7154}$

b) If six adult males are randomly selected, what is the probability that their average height is between 66.0 inches and 72.0 inches? (19 points)

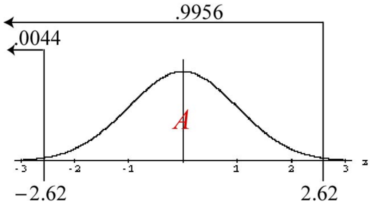
Let $\overline{X}$ be the mean height in a group of six randomly selected adult males.
We can apply the Central Limit Theorem (CLT), because the original distribution is approximately normal.

$$\overline{X} \overset{\overset{\text{approx.}}{\frown}}{\sim} N\left(\mu_{\overline{X}} = \mu = 69.0,\ \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{2.8}{\sqrt{6}} \approx 1.143\right)$$

If $\overline{x} = 72.0$ in, then: $z = \dfrac{\overline{x}-\mu_{\overline{X}}}{\sigma_{\overline{X}}}$ or $\dfrac{\overline{x}-\mu}{\sigma/\sqrt{n}} \approx \dfrac{72.0-69.0}{1.143} \approx \dfrac{3.0}{1.143} \approx 2.62$

If $\overline{x} = 66.0$ in, then: $z \approx -2.62$ by symmetry about the mean.

Therefore, $P\left(66.0 \text{ in} < \overline{X} < 72.0 \text{ in}\right) \approx P\left(-2.62 < Z < 2.62\right)$.



$A \approx .9956 - .0044 \approx \boxed{.9912}$. This is higher than the .7154 we obtained in part a); this is due to the tighter clustering of the $\overline{X}$ distribution about the mean in part b).

7) (25 points). According to a CNN poll taken after the first presidential debate of 2012, 25% of debate watchers thought that President Obama won. Let's say 360 debate watchers are randomly selected. Based on the poll results, find the probability that more than 100 of those selected individuals thought that President Obama won. Use a normal approximation to a binomial distribution, and <u>use a continuity correction</u>. Show why the normal approximation is appropriate based on the rules given in class.

- Define "a success on one trial" as "the debate watcher representing the trial thought that President Obama won."
- Then, $p = 0.25$, and $q = 1 - p = 0.75$.
- Let X be the number of successes among the 360 debate watchers. Then, $X \sim \text{Bin}\left(n = 360,\ p = 0.25\right)$.
- Show why the normal approximation is appropriate:

$$np = (360)(0.25) = 90 \geq 5 \quad (\text{Good!})$$

$$nq = (360)(0.75) = 270 \geq 5 \quad (\text{Good!})$$

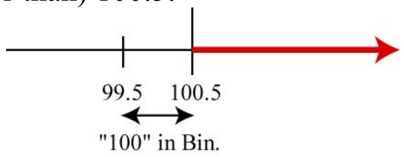
- Therefore, we can use the approximation:

$$X \overbrace{\sim}^{\text{approx.}} N\Big(\mu = np = 90 \text{ voters},$$

$$\sigma = \sqrt{npq} = \sqrt{(360)(0.25)(0.75)} = \sqrt{67.5} \approx 8.216 \text{ voters}\Big)$$

- Apply the continuity correction and go to z scores:

The whole number "100" from the binomial distribution corresponds to the real numbers from 99.5 to 100.5 from the normal distribution. Since "more than" excludes 100, itself, we exclude those numbers but include the numbers to the right of (that is, greater than) 100.5.

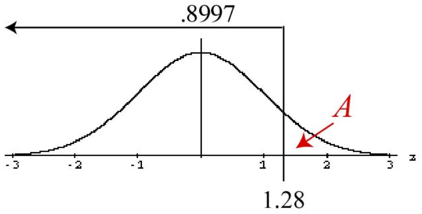


Therefore, $P(X > 100 \text{ voters}) \approx P(X_C > 100.5 \text{ voters})$.

$$\text{If } x_C = 100.5 \text{ voters, then: } z \approx \frac{100.5 - 90}{8.216} \approx \frac{10.5}{8.216} \approx 1.28$$

Therefore, $P(X > 100 \text{ voters}) \approx P(Z > 1.28)$.

- Find the desired probability:



$A \approx 1 - .8997 \approx \boxed{.1003}$