QUIZ 3 SOLUTIONS

(LESSONS 19-23: CONTINUOUS PROBABILITY)
STAT C1000 – SPRING 2026 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate. You do not have to reduce fractions. For example, 10/20 does not have to be rewritten as ½. Do not leave decimal parts in fractions or fractions in fractions, such as .1/.2 or (1/3)/(2/3).

1) (2 points). If $X \sim \text{Uniform}[0,1]$, find $P(0.5 < X < 0.9)$.
You may write your answer as a decimal, percent, or fraction.

$$P(0.5 < X < 0.9) = x_2 - x_1 = 0.9 - 0.5 = \boxed{0.4, \text{ or } 40\%}$$

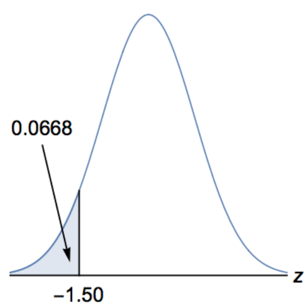
2) (4 points). If $X \sim \text{Uniform}[500, 1000]$, find $P(600 < X < 800)$.
You may write your answer as a decimal, percent, or fraction.

$$P(600 < X < 800) = \frac{x_2 - x_1}{b - a} = \frac{800 - 600}{1000 - 500} = \boxed{\frac{200}{500}, \text{ or } \frac{2}{5}, \text{ or } 0.4, \text{ or } 40\%}$$

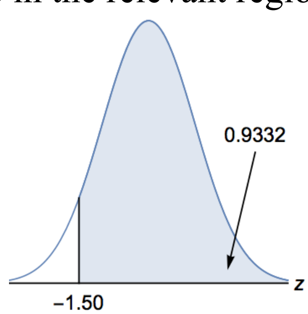
Write *z* scores out to two decimal places. Write probabilities to four decimal places.

3) (17 points). Software tells you that $P(Z < -1.50) \approx 0.0668$.

- a) Sketch a figure clearly showing this fact, as in class. Shade in the relevant region. (3 points)



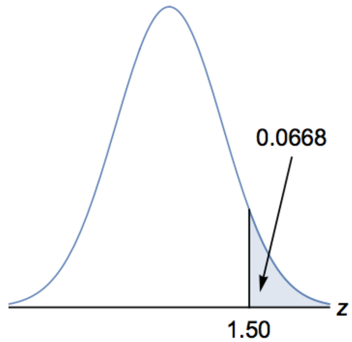
- b) Find $P(Z > -1.50)$. Explain why (show work) and sketch a figure clearly showing this, as in class. Shade in the relevant region. (7 points)



$$P(Z > -1.50) \approx 1 - P(Z < -1.50) \approx 1 - 0.0668 \approx \boxed{0.9332}$$

• c) Find $P(Z > 1.50)$. Explain why and sketch a figure clearly showing this, as in class. Shade in the relevant region. (7 points)

$P(Z > 1.50) = P(Z < -1.50) \approx \boxed{0.0668}$ by **symmetry** of the Z distribution about the mean, $z = 0$.

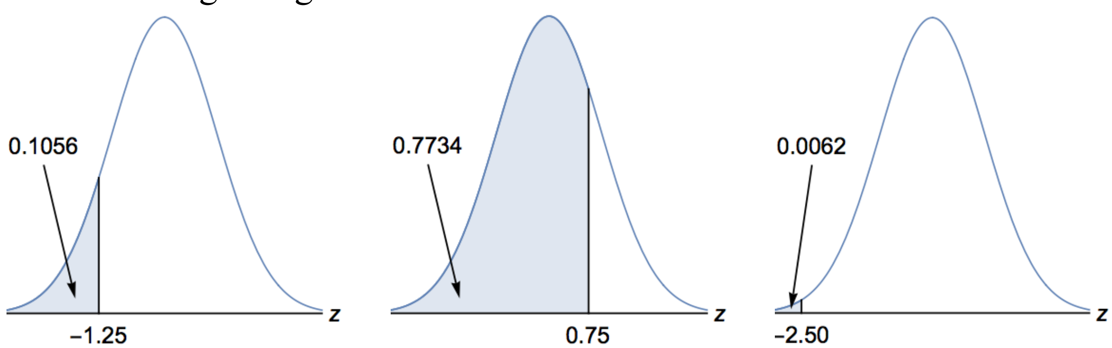


4) (46 points). This information is for parts a) through e).

A large lecture class takes a test. The scores are approximately normally distributed with mean 70 points and standard deviation 8 points. Let X be the score of a randomly selected student in the class.

(Note: The professor gives partial credit so that scores such as 70.1 points, 70.2 points, etc. are possible.)

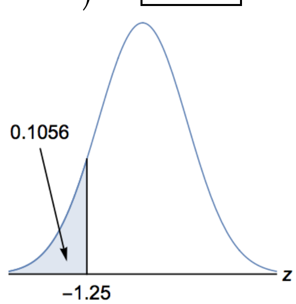
Use these hints regarding the Z distribution:



• a) Find $P(X < 60 \text{ points})$. First write the corresponding **probability expression for** $\boldsymbol{Z}$. Show work by using the Formula for z Scores. (6 points)

$$x = 60 \text{ points} \;\Rightarrow\; z = \frac{x-\mu}{\sigma} = \frac{60-70}{8} \approx -1.25$$

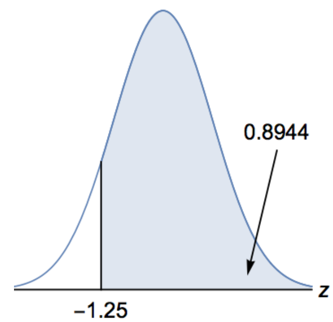
$$P(X < 60 \text{ points}) \approx P(Z < -1.25) \approx \boxed{0.1056}$$



(optional sketch)

• b) Find $P(X > 60 \text{ points})$. First write the corresponding **probability expression for $\boldsymbol{Z}$**. (6 points)

$$P(X > 60 \text{ points}) \approx P(Z > -1.25) \approx 1 - P(Z < -1.25) \approx 1 - 0.1056 \approx \boxed{0.8944}$$



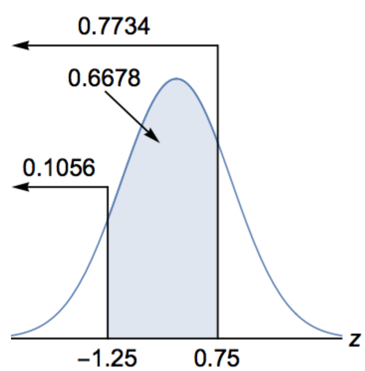
(optional sketch)

• c) Find $P(60 \text{ points} < X < 76 \text{ points})$. First write the corresponding **probability expression for $\boldsymbol{Z}$**. Use the Formula for z Scores when showing work. You may use your work from part a). (8 points)

$$x = 60 \text{ points} \Rightarrow z = \frac{x-\mu}{\sigma} = \frac{60-70}{8} = -1.25 \quad (\text{found in a}))$$

$$x = 76 \text{ points} \Rightarrow z = \frac{x-\mu}{\sigma} = \frac{76-70}{8} = 0.75$$

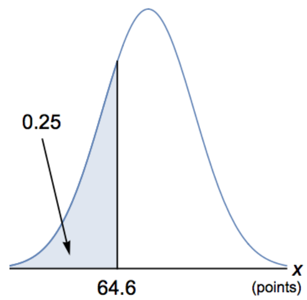
$$P(60 \text{ points} < X < 76 \text{ points}) \approx P(-1.25 < Z < 0.75) \approx 0.7734 - 0.1056 \approx \boxed{0.6678}$$



(optional sketch)

• d) Find the 25th percentile (also known as the 1st quartile) of the distribution of test scores; round it off to the nearest tenth of a point (one decimal place).
Show work by using an appropriate formula.
Write the corresponding **probability statement for X** and **interpret** it, as in class.
Hint: The 25th percentile of the standard normal Z distribution is about -0.67.
(8 points)

$$z \approx -0.67 \Rightarrow x = \mu + z\sigma \approx 70 + (-0.67)(8) \approx \boxed{64.6 \text{ points}}$$



(optional sketch)

The desired probability statement for X is: $\boxed{P(X < 64.6 \text{ points}) \approx 0.25}$.

Interpretation: About 25% of the class scored less than 64.6 points.

• e) Four students who took the test are randomly selected. We will find the probability that the average of their scores $(\overline{X})$ was less than 60 points. That is, we will find $P(\overline{X} < 60 \text{ points})$. We will compare our answer to part a). (18 points)

First write the approximate **sampling distribution** for $\overline{X}$.

$X \overset{\text{approx.}}{\sim}$ Normal, so the Central Limit Theorem (CLT) for Means applies.

$$\overline{X} \overset{\text{approx.}}{\sim} \boxed{N\left(\mu_{\overline{X}} = 70 \text{ points}, \ \sigma_{\overline{X}} = 4 \text{ points}\right)}$$

$$\text{mean, } \mu_{\overline{X}} = \mu = 70 \text{ points}$$

$$\text{SD or SE, } \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{4}} = \frac{8}{2} = 4 \text{ points}$$

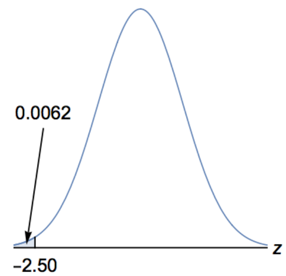
Write the **probability expression for Z** corresponding to $P(\overline{X} < 60 \text{ points})$
Show work by using the Formula for z Scores for Sample Means.

$$\overline{x} = 60 \text{ points} \Rightarrow z = \frac{\overline{x} - \mu_{\overline{X}}}{\sigma_{\overline{X}}} = \frac{60 - 70}{4} = -2.50$$

$$P(\overline{X} < 60 \text{ points}) \approx \boxed{P(Z < -2.50)}$$

Find $P\left(\overline{X} < 60 \text{ points}\right)$.

$$P\left(\overline{X} < 60 \text{ points}\right) \approx P\left(Z < -2.50\right) \approx \boxed{0.0062}$$



(optional sketch)

Compare the result to the one from part a), where we cared about the score of just one random student; is your result here higher or lower than the one in part a)? Remember that we are dealing with tail probabilities "away" from the mean (probability masses in tails that exclude the mean). **Box in one:**

- The result here is **higher** than the one from part a).
- The result here is **lower** than the one from part a).

This is due to the tighter clustering of the sample mean distribution about the original distribution's mean, 70 points. Tail probabilities "away" from the mean shrink.

5) A student answers all the questions on a multiple-choice test with 100 questions. Each question has four possible options: "A," "B," "C," or "D," only one of which is correct. The student guesses randomly on all questions. The random variable X is the number of questions the student gets correct. Approximate the probability that the student gets at least 20 questions correct, $P(X \geq 20)$. Apply an appropriate continuity correction. Follow these steps: (31 points)

- a) **Describe the distribution of X**, as in class. (5 points)

$$\boxed{X \sim \underset{\text{[questions]}}{\text{Bin}}\left(n = 100,\ p = \frac{1}{4} \text{ or } 0.25\right)}$$

- b) **Verify** that a **normal approximation** to the distribution of X would be appropriate, as in class. (4 points)

$$q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}, \text{ or } 0.75$$

$$\boxed{\begin{array}{l} np = (100)\left(\frac{1}{4}\right) = 25 \geq 5 \\ nq = (100)\left(\frac{3}{4}\right) = 75 \geq 5 \end{array}}$$

- c) **Describe the normal distribution** that can be used to approximate the distribution of X, as in class. Round off to five significant figures if necessary. (8 points)

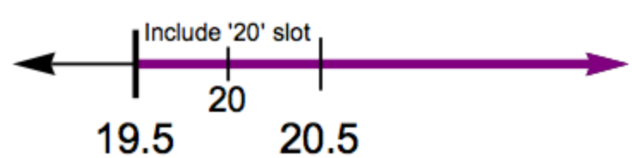
$$\text{mean, } \mu = np = (100)\left(\frac{1}{4}\right) = 25 \text{ [questions]}$$

$$\text{SD, } \sigma = \sqrt{npq} = \sqrt{(100)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)} \approx 4.3301 \text{ [questions]}$$

Therefore:

$$X \overset{\text{approx.}}{\sim} \boxed{\text{N}(\mu = 25,\ \sigma \approx 4.3301)}$$

- d) **Apply a continuity correction** and rewrite $P(X \geq 20)$ in terms of X_c, as in class. You could instead draw a good number-line picture, as in class. (4 points)



$$\boxed{P(X_c \geq 19.5), \text{ or } P(X_c > 19.5)}$$

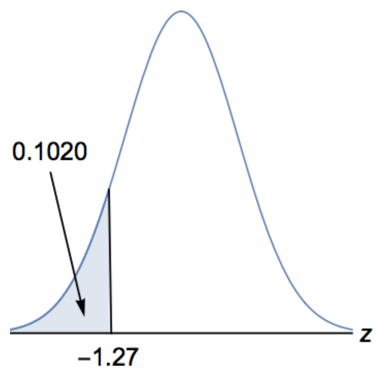
• e) Find the z **score for the boundary value of** x_c; show work by using the Formula for z Scores and round it off to two decimal places. (4 points)

$$x_c = x = 19.5 \quad \Rightarrow \quad z = \frac{x-\mu}{\sigma} \approx \frac{19.5-25}{4.3301} \approx \boxed{-1.27}$$

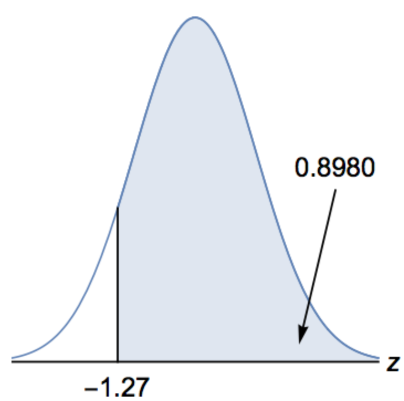
• f) Write the **probability expression for** Z corresponding to your expression for X_c (or picture) from part d), as in class. (2 points)

$$\boxed{P(Z \geq -1.27), \text{ or } P(Z > -1.27)}$$

• g) Approximate $P(X \geq 20)$, as in class. (4 points)
Use these hints regarding the Z distribution:



$$P(Z > -1.27) \approx 1 - P(Z < -1.27)$$
$$\approx 1 - 0.1020$$
$$\approx \boxed{0.8980}$$



(optional sketch)